Lecture 7: BAO, RSD, and the AP Effect (I)

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Abstract

This lecture introduces the main observables of galaxy clustering. We begin with the two-point correlation function and the power spectrum, then explain the physical origins of baryon acoustic oscillations (BAO), the Alcock-Paczynski (AP) effect, and redshift-space distortions (RSD). The main goal is conceptual separation: BAO is a standard-ruler effect, AP is a geometric distortion, and RSD is a velocity distortion.

Learning goals

After this lecture, students should be able to:

- define the galaxy two-point correlation function and power spectrum;
- explain in words what BAO, AP, and RSD each measure;
- understand why BAO is primarily sensitive to expansion history;
- distinguish a geometric anisotropy from a velocity-induced anisotropy;
- describe why anisotropic clustering is so valuable in modern cosmology.

1 The galaxy two-point correlation function

The two-point correlation function measures excess clustering relative to a random Poisson distribution. For galaxies in volume elements $\mathrm{d}V_1$ and $\mathrm{d}V_2$ separated by $\mathbf{r}$,

$$\mathrm{d}P = \bar{n}^2 \left[1 + \xi(\mathbf{r})\right] \mathrm{d}V_1 \mathrm{d}V_2,$$

where $\bar{n}$ is the mean number density.

If $\xi(\mathbf{r}) > 0$, galaxy pairs at separation $\mathbf{r}$ are more common than in a random catalog. If $\xi(\mathbf{r}) = 0$, the distribution is random at that scale.

Isotropic and anisotropic clustering. If the clustering is statistically isotropic, we can write $\xi(\mathbf{r}) = \xi(r)$. In redshift surveys, however, line-of-sight effects make the clustering anisotropic, so we often use

$$\xi(s, \mu),$$

where s is the pair separation and $\mu = \cos\theta$ is the cosine of the angle between the separation vector and the line of sight.

It is then natural to expand the anisotropy in Legendre multipoles:

$$\xi(s, \mu) = \sum_{\ell} \xi_\ell(s)\, \mathcal{L}_\ell(\mu).$$

2 The power spectrum

The power spectrum is the Fourier transform of the correlation function:

$$P(\mathbf{k}) = \int \xi(\mathbf{r})\, \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}\, \mathrm{d}^3 r,$$

up to the chosen Fourier-convention factors. In an isotropic Universe, it reduces to $P(k)$.

The correlation function and the power spectrum contain the same information in the ideal limit. In practice, some effects are easier to model in configuration space and others in Fourier space.

For anisotropic clustering we can also define power-spectrum multipoles through

$$P(k, \mu) = \sum_{\ell} P_\ell(k)\, \mathcal{L}_\ell(\mu).$$

3 Baryon acoustic oscillations

Before recombination, photons and baryons formed a tightly coupled plasma. Gravity pulled matter inward while photon pressure pushed it outward, creating acoustic waves. When photons decoupled from baryons, the waves stopped propagating, leaving a preferred comoving scale: the sound horizon.

The comoving sound horizon at the drag epoch z_d is

$$r_s(z_d) = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)}\, \mathrm{d}z,$$

where the sound speed is

$$c_s(z) = \frac{c}{\sqrt{3(1+R)}}, \qquad R \equiv \frac{3\rho_b}{4\rho_\gamma}.$$

In standard cosmology,

$$r_s(z_d) \approx 147 \text{ Mpc}.$$

How BAO appears in the data. The sound horizon shows up as

- a weak peak in $\xi(r)$ near $r \sim 150$ Mpc;
- oscillatory "wiggles" in the matter or galaxy power spectrum.

Because the physical scale is known, BAO provides a **standard ruler**. Observationally, BAO was first detected in 2005 [1].

Radial and transverse BAO. At redshift z, the same physical ruler can be seen in two directions:

$$\Delta\theta \approx \frac{r_s}{D_M(z)}, \qquad \Delta z \approx \frac{H(z)\, r_s}{c},$$

where $D_M(z)$ is the transverse comoving distance. This is why BAO can constrain both distances and expansion rates.

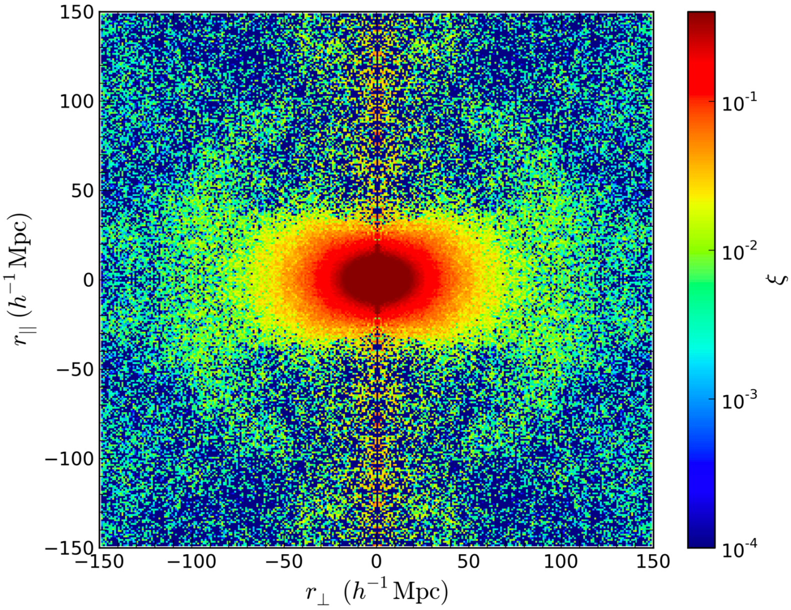

Figure 1: The two-dimensional correlation function measured from BOSS galaxies, showing anisotropic clustering in the transverse and line-of-sight directions [4].

4 The Alcock-Paczynski effect

To convert observed angles and redshifts into comoving distances, we must assume a fiducial cosmology. If that cosmology is incorrect, an intrinsically isotropic pattern appears distorted.

This is the Alcock-Paczynski effect. It is a **geometric** anisotropy, not a dynamical one.

A common parameterization is

$$\alpha_{\perp} = \frac{D_M(z)\, r_s^{\rm fid}}{D_M^{\rm fid}(z)\, r_s}, \qquad \alpha_{\parallel} = \frac{H^{\rm fid}(z)\, r_s^{\rm fid}}{H(z)\, r_s}.$$

If the fiducial cosmology is correct, then $\alpha_{\perp} = \alpha_{\parallel} = 1$.

The AP effect [3] is therefore a pure geometric probe of the expansion history through measurements of $D_M(z)$ and $H(z)$.

5 Redshift-space distortions: first look

Observed galaxy redshifts contain both the cosmological redshift and the Doppler shift from the galaxy's peculiar velocity. Therefore, when we infer the line-of-sight position from redshift, galaxy clustering is distorted.

This is redshift-space distortion. Unlike AP, it is not caused by assuming the wrong background cosmology. It is caused by **peculiar velocities** generated by gravitational growth.

Why RSD is useful. Because peculiar velocities are sourced by gravity, RSD gives access to the growth rate of structure. In practice, this is often summarized by observables such as $f\sigma_8$.

6 Redshift-space distortions: linear theory

RSD is another special three-dimensional clustering pattern of galaxies, but it is due to local motions of galaxies under gravity [2]. If galaxies only co-moved with the cosmic expansion and had no local peculiar velocities, clustering would remain isotropic. In reality, galaxies fall toward overdense regions, so their observed redshifts include both Hubble expansion and line-of-sight peculiar motion. Since positions in a redshift survey are inferred from redshift, these motions distort the apparent clustering pattern.

On sufficiently large scales, the distortion is described by the Kaiser formula:

$$P^s(k,\mu) = \left(1 + \beta\mu^2\right)^2 P^r(k),$$

where P^s and P^r are the redshift-space and real-space power spectra, respectively, μ is the cosine of the angle to the line of sight, and

$$\beta = \frac{f}{b}.$$

Here f is the linear growth rate and b is the galaxy bias. This makes clear why RSD is so useful for testing gravity and measuring structure growth.

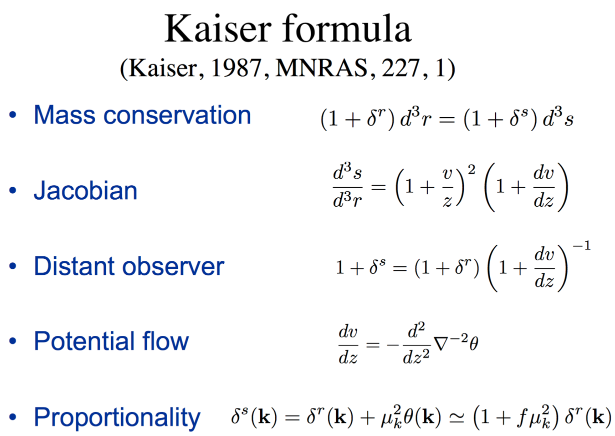

Figure 2: Schematic illustration of the Kaiser effect. Large-scale coherent infall makes structures appear squashed along the line of sight in redshift space.

7 BAO, AP, and RSD: conceptual separation

Students often mix these three effects together. The cleanest summary is:

Effect	Physical origin	Main cosmological use
BAO	Frozen sound horizon from the early photon-baryon plasma	Standard ruler; geometry and expansion history
AP	Wrong conversion from angles and redshifts to distances	Purely geometric test of $D_M(z)$ and $H(z)$
RSD	Peculiar velocities along the line of sight	Growth of structure and tests of gravity

8 Summary

Large-scale galaxy clustering contains multiple layers of information:

- the overall clustering amplitude and shape;
- the BAO standard ruler;

- AP anisotropy from geometry;
- RSD anisotropy from velocities.

The power of modern redshift surveys comes from measuring all of these simultaneously.

Suggested reading

- Dodelson and Schmidt, chapter on large-scale structure.
- White's BAO review notes for intuition about the sound horizon.
- Review articles on anisotropic clustering analyses.

Homework

1. **Correlation function.** Starting from

$$\mathrm{d}P = \bar{n}^2[1 + \xi(\mathbf{r})]\mathrm{d}V_1\mathrm{d}V_2,$$

   explain in words what it means for $\xi(r)$ to be positive, zero, or negative.

2. **Multipole expansion.** Show that if $\xi(s, \mu)$ is symmetric under $\mu \to -\mu$, then only even Legendre multipoles appear in the expansion.

3. **BAO angular scale.** Assume a flat ΛCDM model with $\Omega_\mathrm{m} = 0.3$, $\Omega_\Lambda = 0.7$, and $D_M(z = 0.8) = 2800$ Mpc. Estimate the BAO angular scale $\Delta\theta$ using $r_s = 147$ Mpc.

4. **BAO radial scale.** Using the same cosmology, suppose $H(z = 0.8) = 109\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. Estimate the BAO redshift interval Δz.

5. **Conceptual comparison.** In one page or less, explain the physical difference between BAO, AP, and RSD. Your answer should state clearly which effect is mainly sensitive to geometry and which effect is mainly sensitive to growth.

6. **Optional coding task.** Make a sketch or simple plot showing how an intrinsically circular feature would look after an AP distortion with $\alpha_\perp \neq \alpha_\parallel$.

References

[1] D. J. Eisenstein *et al.* [SDSS Collaboration], "Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies," Astrophys. J. **633**, 560 (2005), doi:10.1086/466512, [astro-ph/0501171].

[2] N. Kaiser, "Clustering in real space and in redshift space," Mon. Not. Roy. Astron. Soc. **227**, 1 (1987).

[3] C. Alcock and B. Paczynski, "An evolution free test for non-zero cosmological constant," Nature **281**, 358 (1979), doi:10.1038/281358a0.

[4] L. Samushia *et al.*, "The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: measuring growth rate and geometry with anisotropic clustering," Mon. Not. Roy. Astron. Soc. **439**, no. 4, 3504 (2014), doi:10.1093/mnras/stu197, [arXiv:1312.4899 [astro-ph.CO]].