LECTURE 17: DARK ENERGY AND MODIFIED GRAVITY

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Learning goals

After this lecture, students should be able to:

- distinguish the two broad explanations of cosmic acceleration: a new stress-energy component in GR, or a modification of gravity on cosmological scales;
- derive how a general dark-energy equation of state changes $H(a)$ and why the CPL form $w(a) = w_0 + w_a(1 - a)$ is used in current data analyses;
- explain the difference between geometry probes, such as SNe and BAO, and growth probes, such as RSD, weak lensing, clusters, and full-shape clustering;
- summarize the recent DESI and DES results in a balanced way: current combinations show an intriguing preference for $w_0 > -1$, $w_a < 0$, while tests of gravitational growth remain broadly consistent with GR;
- understand what MGCAMB changes relative to CAMB, and how $\mu(k, a)$, $\gamma(k, a)$, and $\Sigma(k, a)$ enter cosmological observables.

1. Cosmic Acceleration: Recap

Late–1990s Type Ia supernovae (SNe Ia) established that the expansion rate $\dot{a} > 0$ is accelerating [1, 2]. Independent evidence from CMB anisotropies, baryon–acoustic oscillations (BAO), and large-scale structure (LSS) firmly corroborates this picture. Within General Relativity (GR) the Friedmann equation reads

$$H^2 = H_0^2\big[\Omega_\mathrm{m}a^{-3} + \Omega_\mathrm{r}a^{-4} + \Omega_k a^{-2} + \Omega_\mathrm{DE}\, e^{-3\int_1^a [1+w_\mathrm{DE}(x)]\mathrm{d}x/x}\big], \tag{1}$$

so $\ddot{a} > 0$ demands either a component with $w_\mathrm{DE} < -1/3$ (*dark energy*) or a breakdown of GR on cosmic scales (*modified gravity*).

2. Dark-Energy Models

This section lays out the most widely studied classes of dark energy (DE) with their Lagrangians, stress–energy tensors, and equations of motion. Throughout we set $c = \hbar = 1$ and adopt the $(-, +, +, +)$ metric signature.

2.1. Cosmological Constant Λ.

Lagrangian. A true constant vacuum energy is represented by

$$\mathcal{L}_\Lambda = -\rho_\Lambda, \tag{2}$$

so that $p_\Lambda = -\rho_\Lambda$ giving $w_\mathrm{DE} = -1$.

Stress–energy tensor. $T^{\mu}_{\nu} = -\rho_{\Lambda}\,\delta^{\mu}_{\nu}$, which is covariantly conserved automatically.
Dynamics. There is no dynamical field—ρ_{Λ} is constant and appears as a source term in the Friedmann equations.

2.2. **Canonical Quintessence ($-1 < w_{\rm DE} < 1$).**
Lagrangian.

$$\mathcal{L}_{\phi} = \frac{1}{2}\partial_{\mu}\phi\,\partial^{\mu}\phi - V(\phi). \tag{3}$$

Energy density and pressure.

$$\rho_{\phi} = \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad p_{\phi} = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{4}$$

Thus $w_{\rm DE} = (\dot{\phi}^2 - 2V)/(\dot{\phi}^2 + 2V)$ with $w_{\rm DE} > -1$ when the kinetic term is positive.
Equation of motion. Varying $\mathcal{L}_{\phi}$ yields the Klein–Gordon equation in an FRW background:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{5}$$

Slow-roll solutions drive $w_{\rm DE} \to -1$ if $V \gg \dot{\phi}^2$.

2.3. **Phantom Field ($w_{\rm DE} < -1$).**
Lagrangian.

$$\mathcal{L}_{\sigma} = -\frac{1}{2}\partial_{\mu}\sigma\,\partial^{\mu}\sigma - V(\sigma). \tag{6}$$

A negative kinetic term flips the sign of the energy density and pressure contributions.
Energy density and pressure.

$$\rho_{\sigma} = -\frac{1}{2}\dot{\sigma}^2 + V(\sigma), \qquad p_{\sigma} = -\frac{1}{2}\dot{\sigma}^2 - V(\sigma). \tag{7}$$

Consequently $w_{\rm DE} = (-\dot{\sigma}^2 - 2V)/(-\dot{\sigma}^2 + 2V) < -1$. Phantom models violate the null-energy condition and can culminate in a future Big-Rip singularity at $a \to \infty$ in finite cosmic time.

2.4. **Quintom.** A *quintom* model combines one quintessence field ϕ and one phantom field σ so that the effective $w_{\rm DE}$ can cross the 'phantom divide' $w_{\rm DE} = -1$ while remaining free of simple ghost instabilities:

$$\mathcal{L}_{\rm quintom} = \frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi - \frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma - V(\phi,\sigma). \tag{8}$$

Crossing occurs when $p_{\phi} + p_{\sigma} + \rho_{\phi} + \rho_{\sigma} = 0$ at some time.

2.5. ***k*-Essence.**
Lagrangian. A generalised kinetic theory

$$\mathcal{L}_{k} = P(X,\psi), \qquad X \equiv \frac{1}{2}\partial_{\mu}\psi\partial^{\mu}\psi. \tag{9}$$

Stress–energy tensor.

$$T^{\mu}_{\nu} = P_{,X}\partial^{\mu}\psi\partial_{\nu}\psi + P\,\delta^{\mu}_{\nu}. \tag{10}$$

Equation of motion.

$$(P_{,X} + 2XP_{,XX})\,\Box\psi + P_{,X\psi}\partial_\mu\psi\partial^\mu\psi - P_{,\psi} = 0. \tag{11}$$

Propagation speed is $c_s^2 = P_{,X}/(P_{,X} + 2XP_{,XX})$, impacting clustering and ISW effect.

2.6. **Early Dark Energy (EDE).** A small DE fraction at $z \gg 1$ can relieve the H_0 tension.

$$\Omega_{\rm DE}(a) = \frac{\Omega^0_{\rm DE} - f_{\rm EDE}\left[1 - a^{-3(1+w_{\rm DE})}\right]}{\Omega_{\rm m}a^{-3} + \Omega_{\rm r}a^{-4} + \Omega^0_{\rm DE}} + f_{\rm EDE}, \tag{12}$$

with constant $f_{\rm EDE} \lesssim 0.1$.

Model	Lagrangian snippet	$w_{\rm DE}$ range	Key feature
Λ	$-\rho_\Lambda$	-1	Simple; fine-tuning
Quintessence	$\frac{1}{2}\dot\phi^2 - V$	> -1	Slow-roll dynamics
Phantom	$-\frac{1}{2}\dot\sigma^2 - V$	< -1	NEC violation
Quintom	Mix of both	crosses -1	Avoids singular kinetic pole
k-Essence	$P(X,\psi)$	var.	$c_s^2 \neq 1$
EDE	Phenomenological	> -1	Resolves H_0 tension

Table 1. Representative dark-energy models and their properties.

Summary Table.

3. Modified-Gravity Models

We now outline leading modifications to GR that can drive acceleration without dark energy. Actions are in four dimensions unless noted.

3.1. $f(R)$ **Gravity.**
Action.

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,[R + f(R)] + S_{\rm m}[g_{\mu\nu},\Psi]. \tag{13}$$

Field equations.

$$(1+f_R)G_{\mu\nu} - \tfrac{1}{2}fg_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R = 8\pi G T_{\mu\nu}, \tag{14}$$

with scalaron $f_R \equiv \partial f/\partial R$. A *chameleon* mechanism hides fifth forces in dense regions.

3.2. **DGP Braneworld.**
Action (5D)..

$$S = \int_{\rm bulk}\mathrm{d}^5x\sqrt{-g_5}\,\frac{M_5^3}{2}R_5 + \int_{\rm brane}\mathrm{d}^4x\sqrt{-g}\left[\frac{M_4^2}{2}R + \mathcal{L}_{\rm m}\right]. \tag{15}$$

Self-acceleration emerges beyond the crossover scale $r_c = M_4^2/(2M_5^3)$. Non-linearities yield *Vainshtein* screening.

3.3. **Horndeski / Galileon.** Most general scalar–tensor with second-order equations:

$$S = \int \mathrm{d}^4x\sqrt{-g}\sum_{i=2}^{5}\mathcal{L}_i(\phi, X), \qquad X \equiv -\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi. \tag{16}$$

Background functions $(\alpha_K, \alpha_B, \alpha_M, \alpha_T)$ capture deviations; GW170817 implies $\alpha_T \approx 0$ today.

3.4. **dRGT Massive / Bi-gravity.**
Action.

$$S = \frac{M_{\rm Pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\left[R + m_g^2\sum_{n=0}^{4}\beta_n e_n(\sqrt{g^{-1}f})\right] + S_{\rm m}[g, \Psi]. \tag{17}$$

A graviton mass $m_g \sim H_0$ mimics dark energy; bi-gravity promotes $f_{\mu\nu}$ to a dynamical metric.

3.5. **Screening Mechanisms.**
Chameleon: Density-dependent scalar mass (e.g. $f(R)$).
Vainshtein: Derivative self-interactions screen around massive bodies.
K-mouflage: Large kinetic terms shield interactions when $|\partial\phi|$ is large.

Model	Key scale	Screening	GR limit
$f(R)$	$\lvert f_{R0}\rvert \lesssim 10^{-5}$	Chameleon	$f_R \to 0$
DGP	$r_c \approx H_0^{-1}$	Vainshtein	$r \ll r_V$
Horndeski	$\alpha_i(a)$	model-dep.	$\alpha_i \to 0$
Massive g	$m_g \sim H_0$	Vainshtein	$m_g \to 0$

TABLE 2. Representative modified-gravity scenarios.

Summary Table.

4. PHENOMENOLOGICAL PARAMETERISATIONS

CPL Ansatz. The Chevallier-Polarski-Linder (CPL) form is the standard two-parameter description used for late-time dynamical dark energy in many current analyses [3, 4]:

$$w_{\rm DE}(a) = w_0 + w_a(1 - a). \tag{18}$$

Effective-Field-Theory of DE/MG. Perturbation action expanded around FLRW; time-dependent functions map to concrete models (e.g. α_i, μ, γ parameterisation).

Growth Index. Linear growth $f(a) = \Omega_{\rm m}(a)^\gamma$; $\gamma_{\Lambda{\rm CDM}} \approx 0.55$.

5. Connecting Models to Observables

The central pedagogical point is that a dark-energy or modified-gravity model is not tested by w alone. It is tested through a chain of predictions:

(19)
$$\{\text{theory parameters}\} \quad\longrightarrow\quad H(z),\ \chi(z),\ D_A(z),\ D_L(z),\ G(z),\ \Phi{+}\Psi \quad\longrightarrow\quad \text{data vectors.}$$

Here $G(z)$ denotes the linear growth factor, and $\Phi + \Psi$ is the Weyl potential that controls light deflection.

Geometry probes. Type Ia supernovae measure relative luminosity distances,

$$\mu(z) = m_B - M_B = 5\log_{10}\left[\frac{D_L(z)}{10\,\text{pc}}\right], \qquad D_L(z) = (1+z)\chi(z), \tag{20}$$

where M_B is a nuisance parameter calibrated jointly with cosmology. Supernovae are excellent at measuring the shape of the late-time distance-redshift relation, but by themselves they do not fix the absolute distance scale.

BAO measurements provide a standard ruler set by the sound horizon at the baryon drag epoch, r_d. A transverse BAO measurement constrains

$$\frac{D_M(z)}{r_d} = \frac{\chi(z)}{r_d}, \tag{21}$$

while a line-of-sight BAO measurement constrains

$$\frac{D_H(z)}{r_d} = \frac{c}{H(z)r_d}. \tag{22}$$

For an isotropic BAO measurement one often quotes

$$\frac{D_V(z)}{r_d}, \qquad D_V(z) = \left[zD_M^2(z)D_H(z)\right]^{1/3}. \tag{23}$$

Because BAO measurements can be made over a wide redshift range, they are especially powerful for testing whether the dark-energy density evolves with time.

For the CPL parametrisation,

$$w(a) = w_0 + w_a(1-a), \tag{24}$$

energy conservation gives

$$\rho_{\text{DE}}(a) = \rho_{\text{DE},0}\, a^{-3(1+w_0+w_a)} \exp[-3w_a(1-a)]. \tag{25}$$

Equation (25) is the expression actually used in many likelihood calculations. It shows why w_0 and w_a are strongly correlated: distances involve integrals of $H^{-1}(z)$, and $H(z)$ contains an integral of $w(z)$.

Growth probes. Growth probes ask a different question: given the expansion history, do density perturbations grow as GR predicts? In GR with smooth dark energy, the linear matter perturbation approximately obeys

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0, \tag{26}$$

or, using $\ln a$ as the time variable,

$$\delta_m'' + \left(2 + \frac{H'}{H}\right)\delta_m' - \frac{3}{2}\Omega_m(a)\delta_m = 0, \tag{27}$$

where primes denote $d/d\ln a$. Redshift-space distortions measure the velocity field through

$$f(z)\sigma_8(z), \qquad f \equiv \frac{d\ln G}{d\ln a}, \tag{28}$$

while weak lensing measures an integral of the Weyl potential and is often summarized by

$$S_8 \equiv \sigma_8 \left(\frac{\Omega_m}{0.3}\right)^{1/2}. \tag{29}$$

The combination of geometry and growth is what makes late-time cosmology so powerful. A model can fit distances but fail growth, or fit growth but require a distance-redshift relation inconsistent with SNe and BAO.

Modified gravity in the same language. A convenient phenomenological description replaces the GR Poisson and lensing equations by

$$k^2\Psi = -4\pi G a^2\, \mu(k,a)\, \rho_m\Delta_m, \tag{30}$$

$$k^2(\Phi + \Psi) = -8\pi G a^2\, \Sigma(k,a)\, \rho_m\Delta_m. \tag{31}$$

The function μ mainly affects non-relativistic motion and hence RSD and peculiar velocities. The function Σ mainly affects light deflection and hence weak lensing and CMB lensing. GR with negligible anisotropic stress corresponds to $\mu = \Sigma = 1$.

This is a useful place to warn students about a common misconception. A detection of $w(a) \neq -1$ would not automatically imply modified gravity. It could be a new dark-energy component within GR. Conversely, a modified-gravity model can mimic a $w(a)$ background while leaving a different growth history. Therefore the decisive test is a joint fit to distances, velocities, and lensing.

6. Observational Probes as a Consistency Test

The modern data sets can be organized into a simple table.

The phrase "tension" should also be used carefully in class. A mild difference between two data combinations can arise from new physics, underestimated systematics, analysis choices, or statistical fluctuation. The goal of the next few years is not simply to make error bars smaller, but to make the cross-checks redundant enough that the interpretation becomes robust.

Probe	Main observable	What it tests
SNe Ia	$D_L(z)$ up to an unknown absolute calibration	Late-time expansion shape
BAO	$D_M(z)/r_d$, $D_H(z)/r_d$, or $D_V(z)/r_d$	Absolute cosmic ruler and expansion history
CMB	θ_*, $\Omega_m h^2$, $\Omega_b h^2$, $A_s e^{-2\tau}$	Early-universe anchor and long-distance lever arm
RSD/full shape	$f\sigma_8$, AP distortions, shape of $P(k)$	Growth rate, matter density, neutrino mass, gravity
Weak lensing	Shear correlations and S_8	Weyl potential and late-time clustering amplitude
Clusters	Number counts and lensing-calibrated masses	Growth amplitude and volume element

TABLE 3. A compact map from data type to physical information. The most interesting constraints on dark energy and modified gravity come from combinations, not from any one row alone.

7. Latest Constraints (*Planck* 2018 + DESI DR2 + DES Y5 SNe)

For a flat CPL cosmology [12]:

$$w_0 = -0.752 \pm 0.057, \tag{32}$$

$$w_a = -0.86^{+0.23}_{-0.20}. \tag{33}$$

Modified-gravity likelihoods with `MGCAMB` yield [13]

$$\mu_0 = 0.05 \pm 0.22, \tag{34}$$

$$\Sigma_0 = 0.008 \pm 0.045, \tag{35}$$

where μ_0 and Σ_0 parametrize deviations in the Poisson and lensing equations today.

Current data remain consistent with ΛCDM; small hints for weaker growth (σ_8 tension) motivate next-generation surveys.

8. 2026 Update: DESI, DES, and the Current Picture

The most important update for this lecture is that Stage-III data have become precise enough to make the w_0–w_a plane observationally interesting rather than merely illustrative. The emerging picture is subtle. Several combinations involving DESI BAO, supernovae, and CMB data prefer the quadrant

$$w_0 > -1, \qquad w_a < 0, \tag{36}$$

which corresponds to an equation of state that is less negative than -1 today but more negative at earlier late-time epochs. This is often described as a phantom-crossing pattern. At the same time, growth-based tests from full-shape clustering and weak lensing remain broadly consistent with GR. This is why current results should be presented as an intriguing hint rather than a discovery.

8.1. **DESI DR2 BAO: a sharper expansion history.** DESI DR2 BAO uses the first three years of DESI observations and contains BAO measurements from more than 14 million galaxies and quasars, together with Lyman-α forest BAO at high redshift [12, 14]. The main BAO observables are ratios of distances to the drag-epoch sound horizon r_d:

$$\left\{\frac{D_M(z)}{r_d}, \frac{D_H(z)}{r_d}\right\} \quad \text{or} \quad \frac{D_V(z)}{r_d}. \tag{37}$$

DESI DR2 found that the BAO distance-redshift relation is well described by flat ΛCDM on its own, but the BAO-preferred parameters are in mild tension with CMB-inferred parameters. Allowing the CPL form $w(a) = w_0 + w_a(1-a)$ alleviates this difference, with DESI BAO plus CMB preferring dynamical dark energy over ΛCDM at about 3.1σ; with supernovae added, the preference depends on the supernova sample and ranges from about 2.8σ to 4.2σ [12].

For students, the important lesson is not the exact significance number. The lesson is the geometry. BAO gives distances at many redshifts, CMB fixes the early-universe ruler and the distance to recombination, and SNe fill in the low-redshift relative distance curve. If those curves do not all pass through the same ΛCDM point, the CPL plane is the simplest diagnostic plot.

8.2. **DESI full shape and gravity.** BAO intentionally compresses the galaxy distribution to a robust standard-ruler measurement. Full-shape analysis uses more of the clustering information. Schematically, in redshift space one models multipoles of the galaxy power spectrum,

$$P_\ell^g(k,z) = \frac{2\ell+1}{2}\int_{-1}^{1} d\mu\, P_g^s(k,\mu,z) L_\ell(\mu), \tag{38}$$

where, at linear order,

$$P_g^s(k,\mu,z) \simeq \left[b_1(z) + f(z)\mu^2\right]^2 P_m(k,z). \tag{39}$$

Modern analyses add nonlinear bias, redshift-space distortion corrections, counterterms, stochastic terms, survey-window operations, and covariance validation, but Eq. (39) is the clean starting point.

The DESI DR1 full-shape analysis constrained Ω_m, σ_8, neutrino mass, and growth observables, and found that combinations with CMB and DES data continue to prefer the same $w_0 > -1$, $w_a < 0$ region found by BAO-only analyses [15]. A dedicated DESI modified-gravity analysis then fitted μ and Σ parameterisations and found results consistent with GR, for example $\mu_0 = 0.05 \pm 0.22$ and $\Sigma_0 = 0.008 \pm 0.045$ for one combined ΛCDM-background analysis [13]. Thus, in current DESI examples, the possible novelty is more visible in the background expansion sector than in the growth-of-gravity sector.

8.3. **DES Y6: weak lensing, clustering, SNe, BAO, and clusters.** DES is complementary to DESI. DESI is spectroscopic and excels at 3D BAO and full-shape clustering. DES is imaging-based and excels at weak lensing, photometric galaxy clustering, supernovae, photometric BAO, and cluster counts.

The DES Y6 3×2pt analysis combines cosmic shear, galaxy-galaxy lensing, and galaxy clustering over the full DES footprint, using a validated analysis framework with scale cuts, intrinsic-alignment modelling, baryonic-feedback tests, and mock-data validation [17]. It uses about 140 million source-galaxy shapes and about 9 million lens-galaxy positions, and in flat ΛCDM finds

$$S_8 = 0.789 \pm 0.012, \qquad \Omega_m = 0.333^{+0.023}_{-0.028}, \tag{40}$$

with the projected S_8 difference from primary CMB data at roughly 2.6σ [16]. In wCDM, the same analysis gives $w = -1.12^{+0.26}_{-0.20}$, consistent with a cosmological constant within the uncertainty [16].

DES also updated its supernova analysis. The DES-Dovekie reanalysis of the DES five-year SNe sample uses improved photometric cross-calibration, updated white-dwarf calibration, retraining of the SALT3 light-curve model, and other pipeline revisions. When combined with Planck/ACT/SPT CMB and DESI DR2 BAO in flat $w_0 w_a$CDM, it reports

$$w_0 = -0.803 \pm 0.054, \qquad w_a = -0.72 \pm 0.21, \tag{41}$$

with the preference against ΛCDM reduced from the original DES-SN5YR value to about 3.2σ [18]. This is a good example to show students why calibration and analysis choices matter even when the qualitative conclusion is stable.

8.4. **DES multi-probe dynamical dark energy.** A particularly useful 2026 example is the DES multi-probe dynamical dark-energy analysis, which combines growth and geometry from the full DES data set [19]. With DES data alone, using 3×2pt, SNe, and DES BAO, the analysis obtains

$$w_0 = -0.84^{+0.10}_{-0.10}, \qquad w_a = -0.44^{+0.60}_{-0.55}, \tag{42}$$

about 2.2σ away from a cosmological constant. Adding DESI DR2 BAO gives the strongest low-redshift-only test reported in that work,

$$w_0 = -0.84^{+0.06}_{-0.07}, \qquad w_a = -0.53^{+0.33}_{-0.28}, \tag{43}$$

about 2.3σ away from ΛCDM. Adding primary CMB information from Planck, ACT, and SPT gives

$$w_0 = -0.82^{+0.05}_{-0.05}, \qquad w_a = -0.63^{+0.21}_{-0.18}, \tag{44}$$

about 3.0σ away from a cosmological constant [19]. The best-fit region remains $w_0 > -1$ and $w_a < 0$, matching the broad DESI trend.

Data combination	Representative result	Teaching point
DESI DR2 BAO + CMB	CPL preferred over ΛCDM at about 3.1σ	BAO plus CMB creates a strong geometric lever arm
DES-Dovekie SNe + DESI DR2 BAO + CMB	$w_0 = -0.803 \pm 0.054$, $w_a = -0.72 \pm 0.21$	Improved SN calibration lowers but does not erase the evolving-DE preference
DES Y6 3×2pt	$S_8 = 0.789 \pm 0.012$ in ΛCDM	Weak lensing and clustering test growth, not only distances
All DES + DESI BAO + CMB	$w_0 = -0.82 \pm 0.05$, $w_a = -0.63^{+0.21}_{-0.18}$	Multi-probe constraints remain in the $w_0 > -1$, $w_a < 0$ quadrant
DESI full-shape MG	μ_0, Σ_0 consistent with GR	Current growth data do not yet require modified gravity

TABLE 4. Representative recent results to discuss in lecture. The precise numbers will evolve as analyses mature, but the conceptual lesson is robust: geometry hints at evolving dark energy, while growth tests have not yet forced a departure from GR.

8.5. **How to explain the significance to students.** It is tempting to say that DESI or DES has "discovered evolving dark energy". That is too strong. A better classroom phrasing is:

> Current combinations of DESI BAO, DES supernovae, DES weak-lensing/clustering information, and CMB data show a coherent preference in the CPL plane for $w_0 > -1$ and $w_a < 0$. The significance is interesting, typically at the few-sigma level depending on data choices, but it is not yet at the discovery level and remains sensitive to cross-calibration, data combinations, and possible systematics.

This statement helps students separate physical interpretation from statistical excitement.

9. Numerical Tools

`MGCAMB` — **Hands-on Guide.** `MGCAMB` [7, 8, 9, 10] is a branch of `CAMB` that solves the linear Einstein–Boltzmann equations in the presence of modified growth, parameterising deviations from GR through two functions $\mu(k, a)$ and $\gamma(k, a)$ (or equivalently Σ). It outputs CMB power spectra, matter power spectra, ISW, lensing potentials, and growth functions consistent with MG/DE models.

Field equations. MGCAMB implements the scale- and time-dependent modifications via

$$k^2\Psi = -4\pi G a^2\, \mu(k, a)\, \rho\Delta, \tag{45}$$

$$\Phi = \gamma(k, a)\, \Psi, \tag{46}$$

where μ rescales the effective Newton constant and γ encodes the gravitational slip.

Scale-independent form. The default ansatz in many likelihoods sets

$$\mu(k,a) = 1 + \mu_0 \frac{\Omega_{\rm DE}(a)}{\Omega_{\rm DE,0}}, \tag{47}$$

$$\gamma(k,a) = 1 + \gamma_0 \frac{\Omega_{\rm DE}(a)}{\Omega_{\rm DE,0}}, \tag{48}$$

so deviations grow in proportion to the dark-energy density fraction. An alternative uses $\Sigma = \frac{1}{2}\mu(1+\gamma)$ for lensing, with μ_0 and Σ_0 as the free parameters.

A. Obtaining and Compiling.

(1) **Clone the repository:**

```
git clone --recursive https://github.com/sfu-cosmo/MGCAMB.git
cd MGCAMB/fortran/
make
```

An executable `camb` appears in `./bin`.

B. Parameter-file Anatomy. Open `inifiles/params_mg.ini`. The MG block looks like

```
# --- Modified gravity block ---
use_MG              = T
MG_parametrization  = 4
kbin_start          = 0
MG_mu0              = 0.04
MG_sigma0           = 0.10
MG_nu2              = 0.0
MG_growth_only      = F
# ------------------------------
```

Common presets:

Scale-independent: Set `MG_parametrization = 4` and vary (μ_0, Σ_0); adequate for current data.

Scale-dependent: Choose `MG_parametrization = 2` and provide arrays `MG_mu_k`, `MG_Sigma_k` for the desired k-bins.

EFTCAMB bridge: With flag 3, `MGCAMB` reads time-series of $\alpha_i(a)$ generated by `EFTCAMB`.

C. Running `camb`. Execute

```
./bin/camb inifiles/params_mg.ini
```

Key outputs:

```
_outputs/
  camb_matterpower.dat
  camb_lensedCls.dat
  camb_background.dat
```

D. MCMC with `Cobaya`. Create `cobaya_mg.yaml`:

```
params:
  Omega_b: {prior: {min: 0.04, max: 0.06}}
  Omega_c: {prior: {min: 0.2, max: 0.4}}
  MG_mu0:  {prior: {min: -0.5, max: 0.5}}
  MG_sigma0: {prior: {min: -0.5, max: 0.5}}
  # ...

sampler:
  mcmc:
    max_tries: 5e4
    Rminus1_stop: 0.01

theory:
  camb:
    stop_at_error: True
    extra_args:
      use_MG: True
      MG_parametrization: 4

likelihood:
  planck_2018_highl_plik.TTTEEE: null
  desi_y1_bao: null
```

Run

```
cobaya-run cobaya_mg.yaml -r chains/darkside
```

Chains are stored in `chains/darkside_1.txt` and visualised with `GetDist`.

E. Pro-tips.

- **Speed**: disable B-mode lensing and high-ℓ polarisation if only background+growth are needed.
- **Validation**: ensure `use_MG = F` reproduces GR spectra to better than 0.1 %.
- **Cross-checks**: compare (μ_0, Σ_0) posteriors with a growth-index γ analysis.
- **Docker**: ready-made image `ghcr.io/mgcamb-devs/mgcamb:latest` avoids compile issues.

Other Suites.

- `EFTCAMB` / `EFTCosmoMC` for EFT of DE.
- `hi_class` (extension of `CLASS`) covering Horndeski.
- `MontePython`, `Cobaya`—samplers interfacing CAMB/CLASS variants.

10. How to Interpret These Results Physically

The recent DESI and DES examples motivate three different interpretations. They are not mutually exclusive at the level of phenomenology, but they imply different next steps.

1. Dynamical dark energy within GR. The simplest interpretation of $w(a) \neq -1$ is a new degree of freedom in the dark sector. A canonical quintessence field can produce $w > -1$, but it cannot cross $w = -1$ without additional structure. The preferred CPL region $w_0 > -1$, $w_a < 0$ often implies $w < -1$ at earlier late-time redshifts and $w > -1$ today. This behavior points students back to the quintom and more general effective-field-theory discussions above.

2. Modified gravity mimicking dark energy. A modified-gravity theory can reproduce the same background expansion as a dark-energy model, but it generally changes the relation between matter, velocities, and lensing. This is why Eq. (31) is central. If future DESI, Euclid, Rubin, Roman, or SKA data find $w(a) \neq -1$ but $\mu = \Sigma = 1$ to high precision, the natural interpretation is closer to dark energy within GR. If they find correlated departures in μ and Σ, the case for modified gravity becomes much stronger.

3. Residual systematics or statistical fluctuation. Cosmology is a precision field, and the data sets are heterogeneous. BAO, SNe, weak lensing, clusters, and CMB data have very different calibrations and systematics. The DES-Dovekie update is a useful example: recalibration reduced the significance of the evolving-DE preference while leaving the qualitative parameter direction similar [18]. The scientifically responsible conclusion is therefore to treat the current hint as a target for confirmation.

11. Exam-Friendly Summary of the Lecture

For the final exam, the most useful conceptual chain is:

$$\text{cosmic acceleration} \to H(z) \text{ and } w(a) \to \text{distances and growth} \to \text{consistency tests of GR.}$$

The cosmological constant is simple and still extremely successful, but it raises deep theoretical questions. Dynamical dark energy makes w time-dependent and is currently an observationally active possibility because of DESI and DES combinations. Modified gravity changes the field equations and can be tested by comparing geometry with growth.

A compact way to remember the probes is: SNe measure relative distances, BAO measures a standard ruler, the CMB anchors the early universe, RSD measures velocities, weak lensing measures the Weyl potential, and clusters measure the abundance of massive halos. A convincing explanation of cosmic acceleration must pass all of these tests at once.

12. Outlook

- **Euclid** (launched in 2023 and now in survey operations), **LSST/Rubin**, **Roman**, **SKA**: sub-percent precision on w_0, w_a; percent-level on μ_0, Σ_0.
- CMB-S4 and LiteBIRD will sharpen lensing and ISW-cross correlations.
- Synergy with gravitational-wave standard sirens (Einstein Telescope) offers independent distance ladder.

13. Summary

Dark energy and modified gravity provide two conceptually distinct avenues to explain acceleration. Observationally they are disentangled by combining expansion-history data with growth-of-structure probes. Numerical tools such as `MGCAMB` enable rapid mapping from theory space to observables, paving the way for stringent tests in the coming decade.

14. Suggested Reading for Review

For background on dark energy and modified gravity, start with the reviews by Copeland, Sami and Tsujikawa [5] and Koyama [6]. For the observational status, read the DESI DR2 BAO cosmology paper [12], the DESI full-shape cosmology and modified-gravity papers [15, 13], the DES Y6 3×2pt cosmology paper [16], the DES-Dovekie supernova reanalysis [18], and the DES multi-probe dynamical dark-energy paper [19]. For numerical implementation, revisit MGCAMB and compare its μ–γ or μ–Σ parametrisation with the equations in this note [7, 8, 9, 10].

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