Solutions to the Homework in Lecture 14
Fundamentals of Observational Cosmology

These notes provide worked solutions to the homework problems in Lecture 14. Throughout, it is useful to remember the Legendre decompositions

$$\mu^0 = \mathcal{L}_0(\mu), \qquad \mu^2 = \frac{1}{3}\mathcal{L}_0(\mu) + \frac{2}{3}\mathcal{L}_2(\mu), \qquad \mu^4 = \frac{1}{5}\mathcal{L}_0(\mu) + \frac{4}{7}\mathcal{L}_2(\mu) + \frac{8}{35}\mathcal{L}_4(\mu).$$

Problem 1. Kaiser multipoles and numerical coefficients

Question. Starting from the Kaiser formula

$$P_g^{s,\mathrm{lin}}(k,\mu) = (b_1 + f\mu^2)^2 P_\mathrm{L}(k),$$

derive the monopole, quadrupole and hexadecapole. Then evaluate the coefficients for $b_1 = 2$ and $f = 0.75$.

Solution. Expand the square:

$$P_g^{s,\mathrm{lin}}(k,\mu) = \left[b_1^2 + 2b_1 f\mu^2 + f^2\mu^4\right] P_\mathrm{L}(k).$$

Now insert the Legendre decompositions of μ^2 and μ^4:

$$\begin{aligned} P_g^{s,\mathrm{lin}}(k,\mu) &= \left[b_1^2 + 2b_1 f\left(\frac{1}{3}\mathcal{L}_0 + \frac{2}{3}\mathcal{L}_2\right) + f^2\left(\frac{1}{5}\mathcal{L}_0 + \frac{4}{7}\mathcal{L}_2 + \frac{8}{35}\mathcal{L}_4\right)\right] P_\mathrm{L}(k) \\ &= \left[\left(b_1^2 + \frac{2}{3}b_1 f + \frac{1}{5}f^2\right)\mathcal{L}_0 + \left(\frac{4}{3}b_1 f + \frac{4}{7}f^2\right)\mathcal{L}_2 + \frac{8}{35}f^2\mathcal{L}_4\right] P_\mathrm{L}(k). \end{aligned}$$

Therefore the non-zero multipoles are

$$\boxed{P_0(k) = \left(b_1^2 + \frac{2}{3}b_1 f + \frac{1}{5}f^2\right) P_\mathrm{L}(k),}$$

$$\boxed{P_2(k) = \left(\frac{4}{3}b_1 f + \frac{4}{7}f^2\right) P_\mathrm{L}(k),}$$

$$\boxed{P_4(k) = \frac{8}{35}f^2 P_\mathrm{L}(k).}$$

Now set $b_1 = 2$ and $f = 0.75 = 3/4$.

For the monopole,

$$P_0(k) = \left[4 + \frac{2}{3}(2)(0.75) + \frac{1}{5}(0.75)^2\right] P_\mathrm{L}(k) = [4 + 1 + 0.1125]\, P_\mathrm{L}(k) = 5.1125\, P_\mathrm{L}(k).$$

For the quadrupole,

$$P_2(k) = \left[\frac{4}{3}(2)(0.75) + \frac{4}{7}(0.75)^2\right] P_{\rm L}(k) = [2 + 0.32142857\ldots]\, P_{\rm L}(k) \approx 2.32143\, P_{\rm L}(k).$$

For the hexadecapole,

$$P_4(k) = \frac{8}{35}(0.75)^2 P_{\rm L}(k) = 0.12857143\ldots\ P_{\rm L}(k) \approx 0.12857\, P_{\rm L}(k).$$

So numerically,

$$\boxed{P_0 \approx 5.1125\, P_{\rm L}, \qquad P_2 \approx 2.32143\, P_{\rm L}, \qquad P_4 \approx 0.12857\, P_{\rm L}.}$$

Problem 2. Fixed-template, ShapeFit, and direct EFT full modelling

Question. In your own words, explain the difference between a standard fixed-template analysis, a ShapeFit analysis, and a direct EFT full-modelling analysis. Which one varies the cosmological model itself at each likelihood step?

Solution. A **standard fixed-template analysis** starts from a broadband power-spectrum shape computed in one fiducial cosmology. During the fit, one usually varies only a limited set of late-time or phenomenological parameters, such as $(\alpha_\perp, \alpha_\parallel)$, $f\sigma_8$, bias amplitudes, FoG parameters, and smooth broadband nuisance terms. The overall shape is therefore largely inherited from the fiducial model rather than recomputed from first principles at each step.

A **ShapeFit analysis** is still a template method, but it is a more flexible one. It keeps a fiducial baseline spectrum and then allows one or a few extra shape-deformation parameters that change the local slope or tilt of the linear spectrum around a pivot scale. In this way it captures more broadband information than the old compressed BAO+RSD template fit, while still being much cheaper than a fully cosmological recalculation.

A **direct EFT full-modelling analysis** goes further. Here the cosmological parameters themselves are sampled, and at each likelihood step the full theory prediction is recomputed from those parameters, together with the EFT nuisance sector (biases, counterterms, stochastic terms, IR resummation, and so on). This is the most direct route to cosmological parameter inference, but it is also the most computationally expensive and requires the heaviest validation.

Therefore, the method that actually varies the underlying cosmological model itself at each likelihood step is

$$\boxed{\text{the direct EFT full-modelling analysis.}}$$

Problem 3. Why doubling k matters so much in EFT counterterms

Question. Consider the schematic EFT counterterm $P_{\rm ctr} \sim -2c_0 k^2 P_{\rm lin}(k)$. If k doubles, by what factor does the prefactor k^2 change? Explain why this immediately suggests that every choice of $k_{\rm max}$ must be validated on mocks.

Solution. If k doubles, then

$$k \to 2k \qquad \Longrightarrow \qquad k^2 \to (2k)^2 = 4k^2.$$

So the prefactor k^2 increases by a factor of

$$\boxed{4.}$$

This matters because EFT counterterms and other missing higher-order corrections grow rapidly toward small scales (large k). Once one pushes $k_{\max}$ upward, the likelihood becomes much more sensitive to precisely the part of the model where perturbation theory is least reliable and nuisance terms must work hardest. In other words, extending the fit range is never "free": it can tighten statistical errors, but it can also amplify modelling errors and parameter biases. Mock catalogues are therefore needed to test whether a chosen $k_{\max}$ still returns unbiased cosmological parameters within the expected theoretical regime.

Problem 4. Why combine pre-reconstruction, post-reconstruction, and bispectrum information?

Question. Give one reason why DESI combines pre-reconstruction full-shape information with post-reconstruction BAO information, and one reason why adding the bispectrum can improve full-shape constraints further.

Solution. A good reason to combine **pre-reconstruction full-shape** information with **post-reconstruction BAO** information is that the two statistics emphasize different parts of the physics. Pre-reconstruction clustering retains broadband shape, redshift-space distortion, and growth information, while post-reconstruction statistics sharpen the BAO feature and therefore measure the acoustic distance scale more cleanly. Using both in one likelihood keeps the growth and RSD information while also benefiting from a cleaner geometric BAO measurement.

A good reason to add the **bispectrum** is that it provides genuinely new information about nonlinear structure formation and bias. In practice, the bispectrum helps break degeneracies among clustering amplitude, growth rate, and nonlinear bias parameters that are difficult to separate with the power spectrum alone. As a result, joint power-spectrum-plus-bispectrum fits can produce tighter and more robust full-shape constraints than power-spectrum-only analyses.