A minicourse of CAMB/CosmoMC

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Why CAMB?

- Very efficient
- Well-structured (modulised) -- easy to tweak and hack
- Well-supported – cosmocoffee.info
- Popular – mainstream numeric tool in cosmology, used by WMAP, Planck, SDSS, etc

Purpose of this course

- Show you how to use CAMB
- Understand the basics of CAMB by linking CAMB equations to Ma & Bertschinger 1996 paper (astro-ph/9506072)
- Show one example of modifying CAMB – make it work for modified gravity
- Guide you how to modify CAMB for your own research purpose

Look into the code…

Analyze the code using

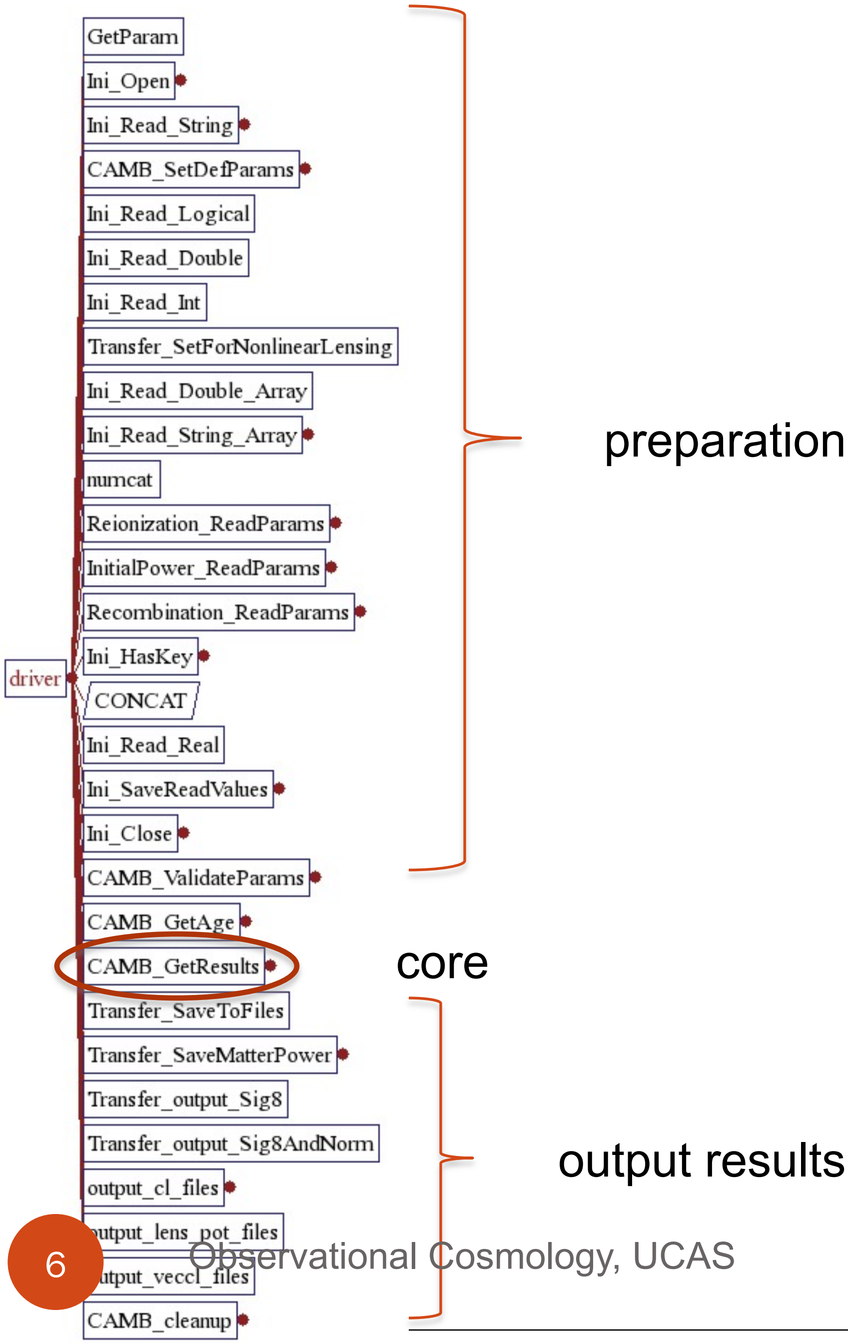
“understand for fortran”

http://www.scitools.com/download/

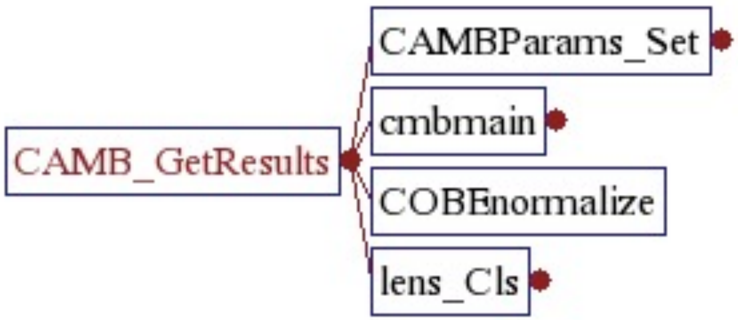
(choose the free 15-day trial version)

Structure of CAMB

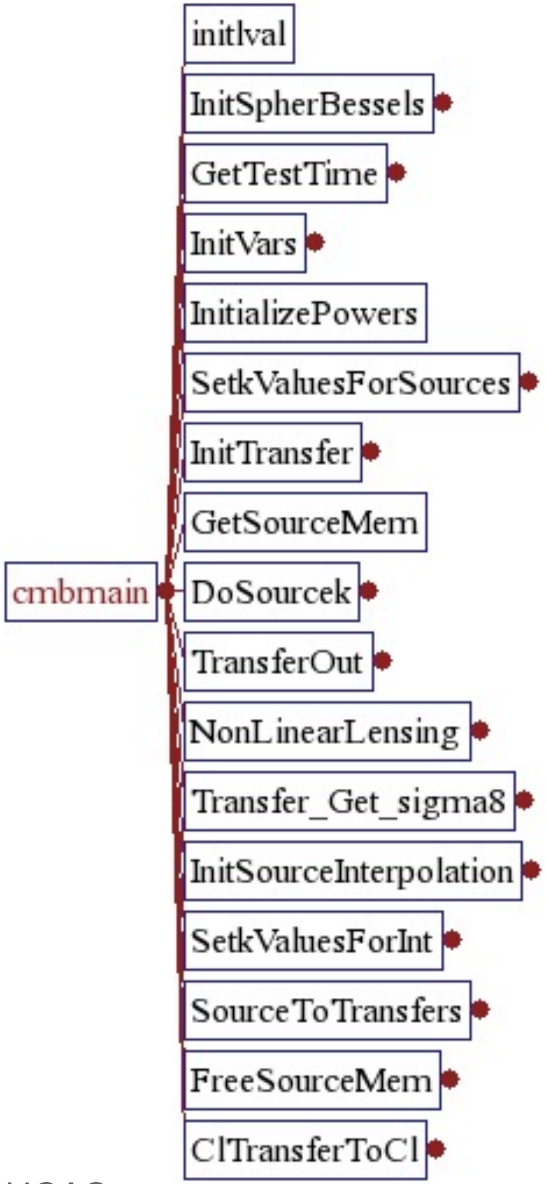
Structure of CAMB



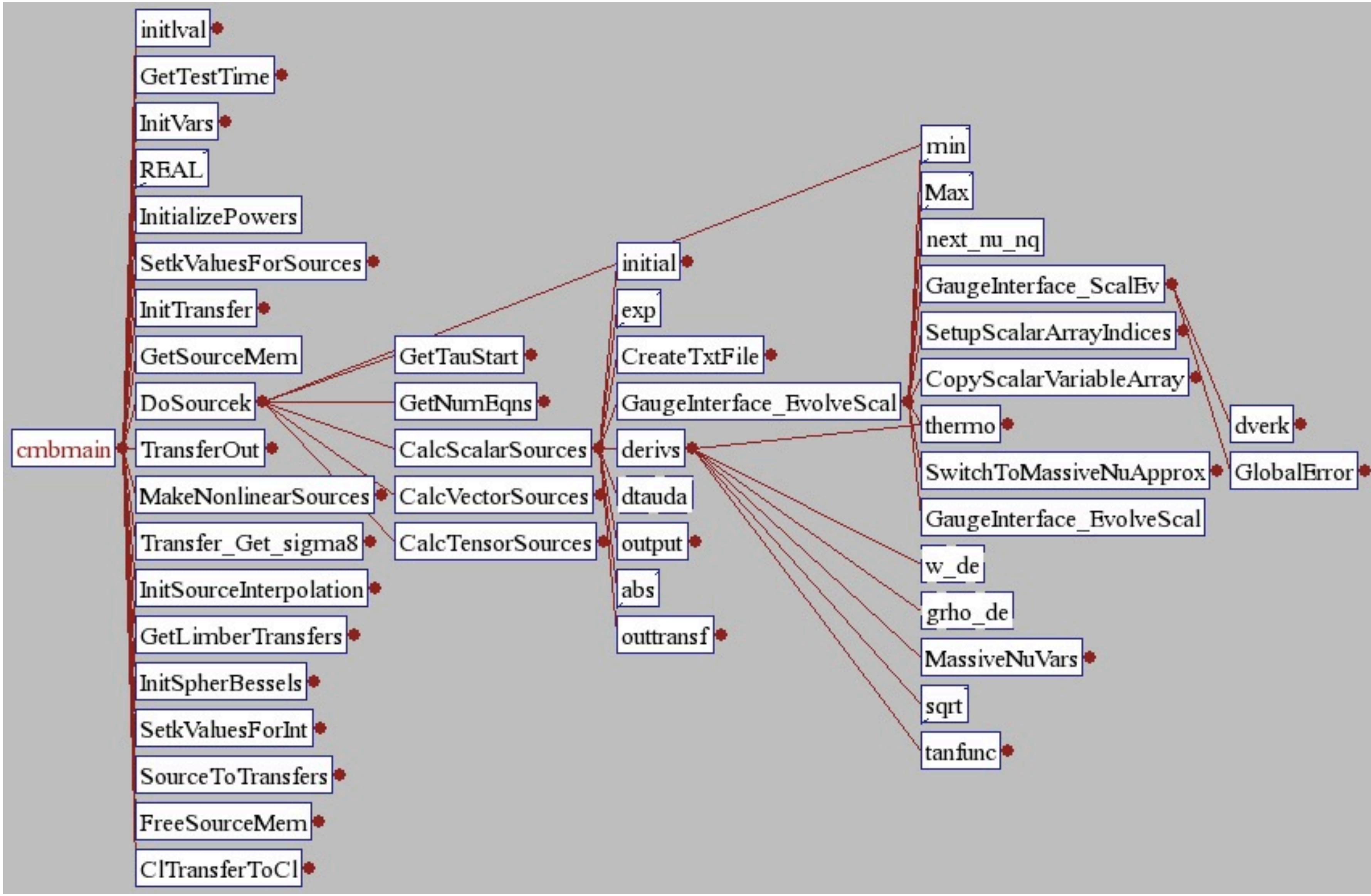
Structure of CAMB_GetResults



Structure of CMBmain



Structure of CMBmain



Initial Conditions

Solve for the coupled differential equations

Do and output statistics (Cls, P(k))

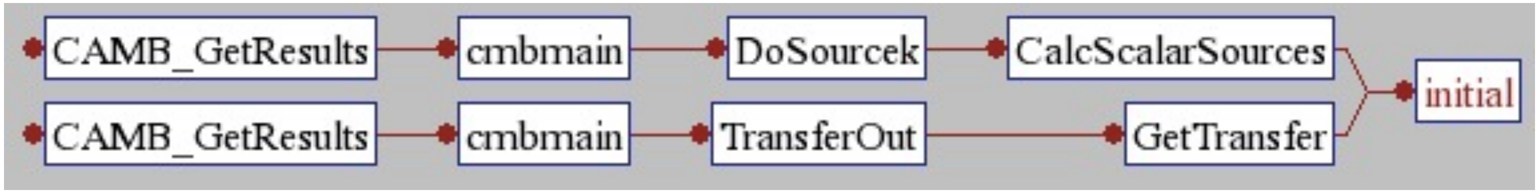
Initial conditions

CAMB notes:

$$\Delta_\gamma = \Delta_\nu = \frac{\beta_2}{3}(k\tau)^2 - \frac{\beta_2}{15}\omega k^2\tau^3$$
$$\Delta_c = \Delta_b = \frac{\beta_2}{4}(k\tau)^2 - \frac{\beta_2}{20}\omega k^2\tau^3$$

CAMB code:

```
x=k*tau
x2=x*x
EV%Kf(1:EV%MaxlNeeded)=1._dl   (if flat)
chi=1 (if flat)
initv(1,i_clxg)=-chi*EV%Kf(1)/3*x2*(1-omtau/5)
initv(1,i_clxr)= initv(1,i_clxg)
initv(1,i_clxb)=0.75_dl*initv(1,i_clxg)
initv(1,i_clxc)=initv(1,i_clxb)
```


CAMB_GetResults
cmbmain
DoSourcek
CalcScalarSources
initial
CAMB_GetResults
cmbmain
TransferOut
GetTransfer

Coupled differential equations

Background

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G a^2 \bar{\rho} - \kappa \, ,$$

$$\frac{d}{d\tau}\left(\frac{\dot{a}}{a}\right) = -\frac{4\pi}{3} G a^2 (\bar{\rho} + 3\bar{P})$$

Linear perturbation in FRW universe

$$ds^2 = -a^2(\eta)[(1+2\Psi(\vec{x},\eta))d\eta^2 - (1-2\Phi(\vec{x},\eta))d\vec{x}^2]$$

$$\nabla_\mu T^{\mu\nu} = 0 \quad \Longrightarrow \quad \begin{aligned} \Phi' &= \frac{1}{3}\left(\delta' + \frac{k}{aH}v\right) \\ \Psi &= \frac{aH}{k}(v' + v) \end{aligned}$$

General Relativity $\Longrightarrow$

$$k^2\Phi = -4\pi G a^2 \rho\delta$$

$$\frac{\Phi}{\Psi} = 1$$

Coupled differential equations

Perturbations

(Perturbed Einstein's equations in synchronous gauge)

$$k^2\eta - \frac{1}{2}\frac{\dot{a}}{a}\dot{h} = 4\pi G a^2\,\delta T^0{}_0(\mathrm{Syn})\,,$$

$$k^2\dot{\eta} = 4\pi G a^2(\bar{\rho} + \bar{P})\theta(\mathrm{Syn})\,,$$

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} - 2k^2\eta = -8\pi G a^2\,\delta T^i{}_i(\mathrm{Syn})\,,$$

$$\ddot{h} + 6\ddot{\eta} + 2\frac{\dot{a}}{a}(\dot{h} + 6\dot{\eta}) - 2k^2\eta = -24\pi G a^2(\bar{\rho} + \bar{P})\sigma(\mathrm{Syn})\,.$$

Coupled differential equations

Energy conservation

$$T^{\mu\nu}{}_{;\mu} = \partial_\mu T^{\mu\nu} + \Gamma^{\nu}{}_{\alpha\beta} T^{\alpha\beta} + \Gamma^{\alpha}{}_{\alpha\beta} T^{\nu\beta} = 0$$

Synchronous gauge:

$$\dot{\delta} = -(1+w)\left(\theta + \frac{\dot{h}}{2}\right) - 3\frac{\dot{a}}{a}\left(\frac{\delta P}{\delta\rho} - w\right)\delta ,$$

$$\dot{\theta} = -\frac{\dot{a}}{a}(1-3w)\theta - \frac{\dot{w}}{1+w}\theta + \frac{\delta P/\delta\rho}{1+w} k^2\delta - k^2\sigma .$$

Coupled differential equations

CAMB language A,B,C

Background: $\text{grho}=8\pi G\rho a^2$, $\text{adotoa}=\dfrac{a'}{a}=\dfrac{\mathrm{d}a/\mathrm{d}\tau}{a}$, tau=conformal time

$\delta T^{\mu\nu}$: $\text{dgrho}=8\pi Ga^2\sum_i \rho_i\delta_i$, $\text{dgq}=8\pi Ga^2\sum_i(\rho_i+p_i)v_i$

$\text{clxc}=\delta_{\mathrm{c}}$, $\text{clxb}=\delta_{\mathrm{b}}$, $\text{clxq}=\delta_{\mathrm{DE}}$

$\delta G^{\mu\nu}$: $\text{etak}=\eta k$, $\text{z}=h'/(2k)$, $\text{sigma}=\dfrac{h'+6\eta'}{2k}$

Coupled differential equations

CAMB code	M+B '96 astro-ph/9506072
$\eta' k = \mathrm{dgq}/2$	$\eta' k^2 = 4\pi G a^2 (\bar{\rho} + \bar{P})\theta$
$\mathrm{clxcdot} = -kz$	$\delta'_c = -\frac{1}{2} h'$
$z = (0.5\mathrm{dgrho}/k + \eta k)/\mathrm{adotoa}$	$k^2\eta - \frac{1}{2}\frac{a'}{a} h' = 4\pi G a^2 \delta T^0_0$
$\mathrm{sigma} = z + 1.5\mathrm{dgq}/k^2$	$\sigma = \frac{h' + 6\eta'}{2k}$

Differential equations to evolve in CAMB

Constraint equations (algebraic)

Baryons

$$\dot{\delta}_b = -\theta_b - \frac{1}{2}\dot{h}\,,$$

```
! Baryon equation of motion.
  clxbdot=-k*(z+vb)
  ayprime(4)=clxbdot
```

$$\dot{\theta}_b = -\frac{\dot{a}}{a}\theta_b + c_s^2 k^2 \delta_b + \frac{4\bar{\rho}_\gamma}{3\bar{\rho}_b} a n_e \sigma_T(\theta_\gamma - \theta_b)\,,$$

```
vbdot=-adotoa*vb+cs2*k*clxb-photbar*opacity*(4._dl/3*vb-qg)
```

Can be simplified in the tightly-coupling limit (homework)

Photons

$$\dot{\delta}_\gamma = -\frac{4}{3}\theta_\gamma - \frac{2}{3}\dot{h}\,,$$

```
! Photon equation of motion
  clxgdot=-k*(4._dl/3._dl*z+qg)
```

$$\dot{\theta}_\gamma = k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) + an_e\sigma_T(\theta_b - \theta_\gamma)\,,$$

```
!Once know slip, recompute qgdot, pig, pigdot
    qgdot = k*(clxg/4._dl-pig/2._dl) +opacity*slip
```

Higher moments, compare to CAMB equations in derivs.f90 (homework)

$$\dot{\delta}_\gamma = -\frac{4}{3}\theta_\gamma - \frac{2}{3}\dot{h} ,$$

$$\dot{\theta}_\gamma = k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) + an_e\sigma_T(\theta_b - \theta_\gamma) ,$$

$$\dot{F}_{\gamma 2} = 2\dot{\sigma}_\gamma = \frac{8}{15}\theta_\gamma - \frac{3}{5}kF_{\gamma 3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\eta}$$

$$-\frac{9}{5}an_e\sigma_T\sigma_\gamma + \frac{1}{10}an_e\sigma_T(G_{\gamma 0} + G_{\gamma 2}) ,$$

$$\dot{F}_{\gamma l} = \frac{k}{2l+1}[lF_{\gamma(l-1)} - (l+1)F_{\gamma(l+1)}] - an_e\sigma_T F_{\gamma l} , \quad l \geq 3 ,$$

$$\dot{G}_{\gamma l} = \frac{k}{2l+1}[lG_{\gamma(l-1)} - (l+1)G_{\gamma(l+1)}]$$

$$+ an_e\sigma_T\left[-G_{\gamma l} + \frac{1}{2}(F_{\gamma 2} + G_{\gamma 0} + G_{\gamma 2})\left(\delta_{l0} + \frac{\delta_{l2}}{5}\right)\right] ,$$

CMB

$$C_l^{XY} \propto \int \frac{dk}{k} \Delta_{\mathcal{R}}^2 I_l^X(k) I_l^Y(k)$$

$$I_l^{X(Y)}(k) = \int \mathcal{S}^{X(Y)}(z) j_l[kr(z)] dz$$

The source term for CMB is in subroutine ‘output’ in equations.f90

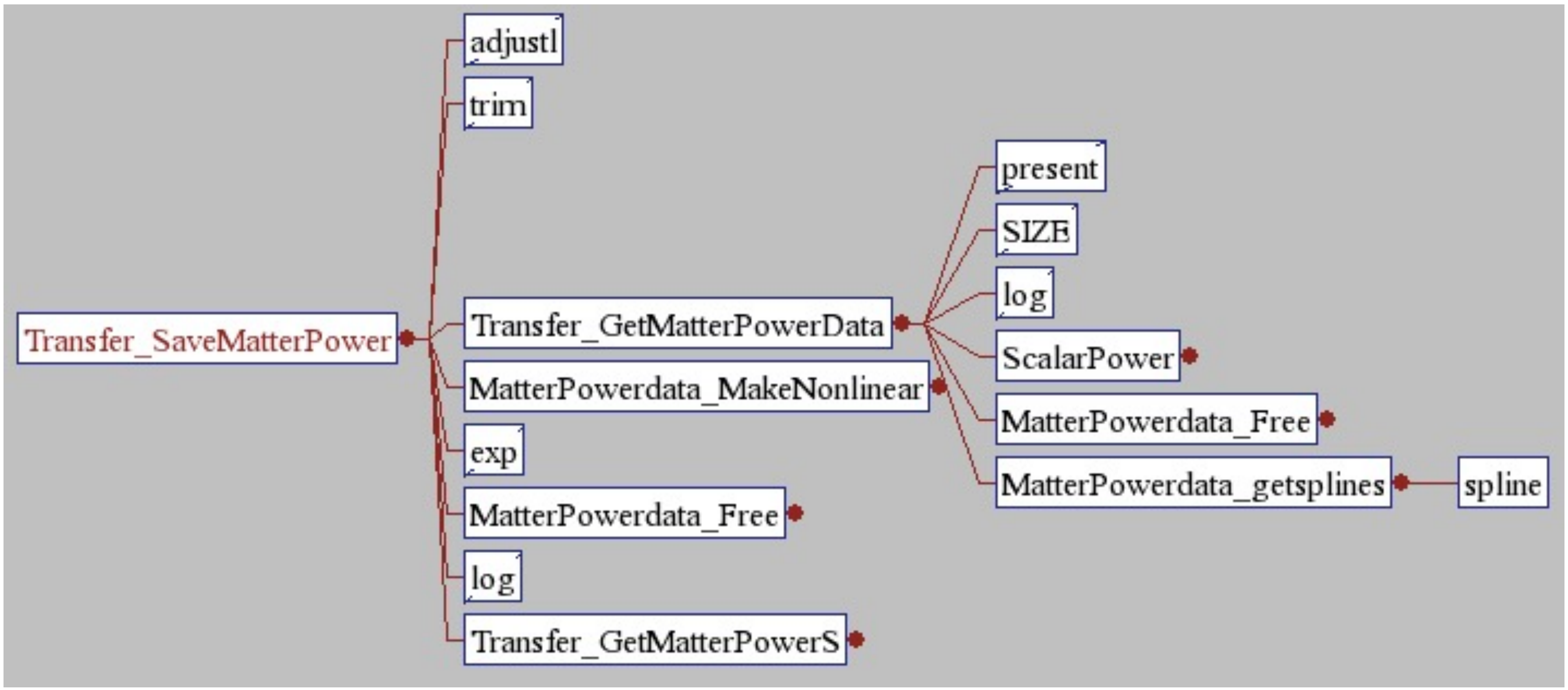
```
 ISW = (4.D0/3.D0*k*EV%Kf(1)*sigma+(-2.D0/3.D0*sigma-2.D0/3.D0*etak/adotoa)*k &
    -diff_rhopi/k**2-1.D0/adotoa*dgrho/3.D0+(3.D0*gpres+5.D0*grho)*sigma/k/3.D0 &
    -2.D0/k*adotoa/EV%Kf(1)*etak)*expmmu(j)

    !e.g. to get only late-time ISW
    !  if (1/a-1 < 30) ISW=0

    !The rest, note y(9)->octg, yprime(9)->octgprime (octopoles)
    sources(1)= ISW +  ((-9.D0/160.D0*pig-27.D0/80.D0*ypol(2))/k**2*opac(j)+ &
    (11.D0/10.D0*sigma- 3.D0/8.D0*EV%Kf(2)*ypol(3)+vb-
9.D0/80.D0*EV%Kf(2)*octg+3.D0/40.D0*qg)/k- &
    (-180.D0*ypolprime(2)-30.D0*pigdot)/k**2/160.D0)*dvis(j) + &
    (-(9.D0*pigdot+
54.D0*ypolprime(2))/k**2*opac(j)/160.D0+pig/16.D0+clxg/4.D0+3.D0/8.D0*ypol(2) + &
    (-21.D0/5.D0*adotoa*sigma-3.D0/8.D0*EV%Kf(2)*ypolprime(3) + &
    vbdot+3.D0/40.D0*qgdot- 9.D0/80.D0*EV%Kf(2)*octgprime)/k + &
    (-9.D0/160.D0*dopac(j)*pig-21.D0/10.D0*dgpi-27.D0/80.D0*dopac(j)*ypol(2))/k**2)*vis(j)
+ &

(3.D0/16.D0*ddvis(j)*pig+9.D0/8.D0*ddvis(j)*ypol(2))/k**2+21.D0/10.D0/k/EV%Kf(1)*vis(j)*
etak
```

But it is understandable!!

Check http://camb.info/theory.html
for the maple files of this source calculated using the famous line-of-sight integral! (homework)

3D Matter power spectrum

$$P_k = \frac{2\pi^2 \mathcal{P}_\chi}{k^3} T_\Delta^2.$$



Useful references

- Camb.info; cosmologist.info/cosmomc/
- Cosmocoffee.info
- M+B: astro-ph/9506072
- Jussi' s ICG lectures: www.icg.port.ac.uk/~valiviij/
- Wayne Hu's tutorials: http://background.uchicago.edu/~whu/
- Numerical Recipe: www.nr.com
- Plotting software: OriginPro, Matlab, IDL, gnuplot
- Icosmology.info