# LECTURE 13: THE MEASUREMENT OF BAO

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## 1. The survey window function

Due to the geometry of the survey volume, we have to convolve the theoretical power spectrum with the survey window function, which is a pain, but there is no way around it.

$$\delta'(\mathbf{x}) = \delta(\mathbf{x})W(\mathbf{x}), \tag{1}$$

$$\tilde{\delta}'(\mathbf{k}) = \tilde{\delta}(\mathbf{k}) * \tilde{W}(\mathbf{k}), \tag{2}$$

where $*$ denotes a convolution.

$$P'(\mathbf{k}) = \int \frac{d^3q}{(2\pi)^3} P(\mathbf{k}-\mathbf{q})|\tilde{W}(\mathbf{q})|^2 \tag{3}$$

Fortunately, it was found that the 3D convolution can be broken into 1D Hankel transformations, due to the convolution theorem [1].

$$P_\ell(k) = 4\pi(-i)^\ell \int \Delta^2 d\Delta \xi_\ell(\Delta) j_\ell(k\Delta) \tag{4}$$

$$P'_\ell(k) = 4\pi(-i)^\ell \left(\frac{2\ell+1}{2q+1}\right) \times A^q_{\ell,\ell'} \int \Delta^2 d\Delta \xi_{\ell'}(\Delta) Q_q(\Delta) j_\ell(k\Delta) \tag{5}$$

$$RR^{\rm tot}_q(\Delta) = \frac{1}{2}\overline{n}_s^2 2\pi\Delta^3 d(\ln\Delta) Q_q(\Delta) \tag{6}$$

$$\xi'_0(\Delta) = \xi_0 Q_0 + \frac{1}{5}\xi_2 Q_2 + \frac{1}{9}\xi_4 Q_4 + \frac{1}{13}\xi_6 Q_6 + \cdots \tag{7}$$

$$\begin{aligned}\xi'_2(\Delta) = \xi_0 Q_2 + \xi_2 & \left(Q_0 + \frac{2}{7}Q_2 + \frac{2}{7}Q_4\right) \\ + \xi_4 & \left(\frac{2}{7}Q_2 + \frac{100}{693}Q_4 + \frac{25}{143}Q_6\right) \\ + \xi_6 & \left(\frac{25}{143}Q_4 + \frac{14}{143}Q_6 + \frac{28}{221}Q_8\right)\end{aligned} \tag{8}$$

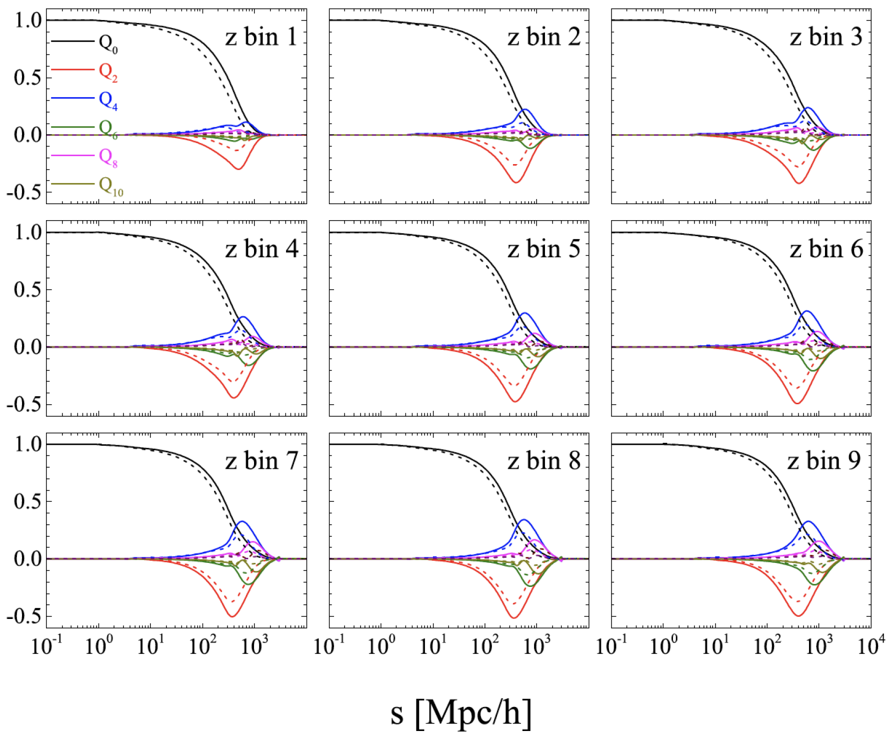


FIGURE 1. The survey window function for BOSS DR12 [6].

## 2. The measurements of BAO

**2.1. BAO using power spectrum multipoles.** The template:

$$\alpha_\perp = \frac{D_A(z) r_d^{\rm fid}}{D_A^{\rm fid}(z) r_d}, \quad \alpha_\parallel = \frac{H^{\rm fid}(z) r_d^{\rm fid}}{H(z) r_d} \tag{9}$$

$$P_g(k,\mu) = P_{\rm nw}(k,\mu)\left\{1 + O(k) e^{-k^2\left[\mu^2\Sigma_\parallel^2 + \left(1-\mu^2\right)\Sigma_\perp^2\right]/2}\right\} \tag{10}$$

$$P_{\rm nw}(k,\mu) = B^2\left(1+\beta\mu^2\right)^2 P_{\rm nw,lin}(k) F(k,\mu) \tag{11}$$

$$F(k,\mu) = \frac{1}{(1+k^2\mu^2\Sigma_s^2/2)} \tag{12}$$

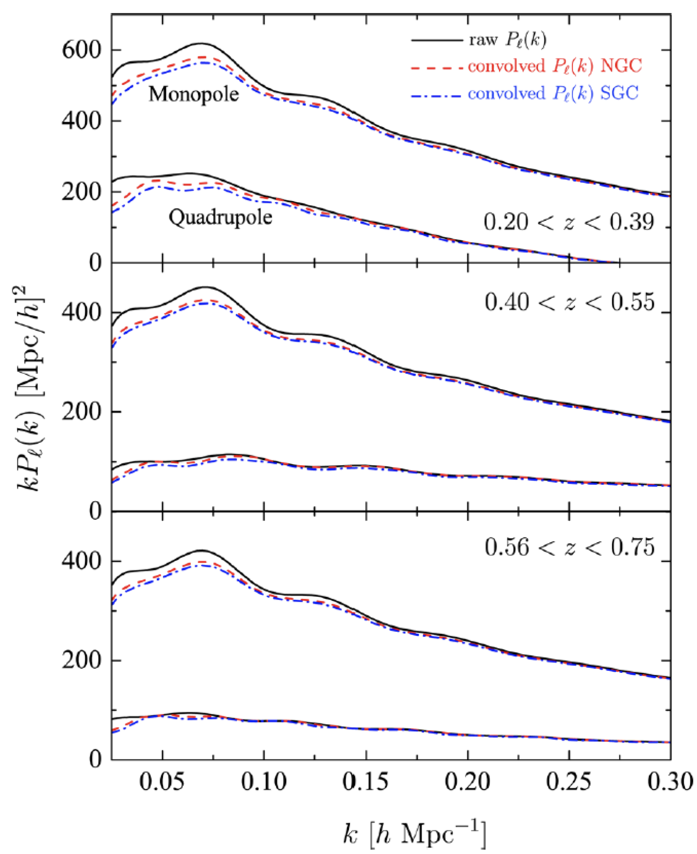


FIGURE 2. The convolved power spectra [6].

$$P_\ell(k) = \left(\frac{r_s^{\rm fid}}{r_s}\right)^3 \frac{2\ell+1}{2\alpha_\perp^2\alpha_\parallel} \int_{-1}^{1} \mathrm{d}\mu P_g\left(k', \mu'\right) \mathcal{L}_\ell(\mu) + \frac{a_{\ell 1}}{k^3} + \frac{a_{\ell 2}}{k^2} + \frac{a_{\ell 3}}{k} + a_{\ell 4} + a_{\ell 5} k$$

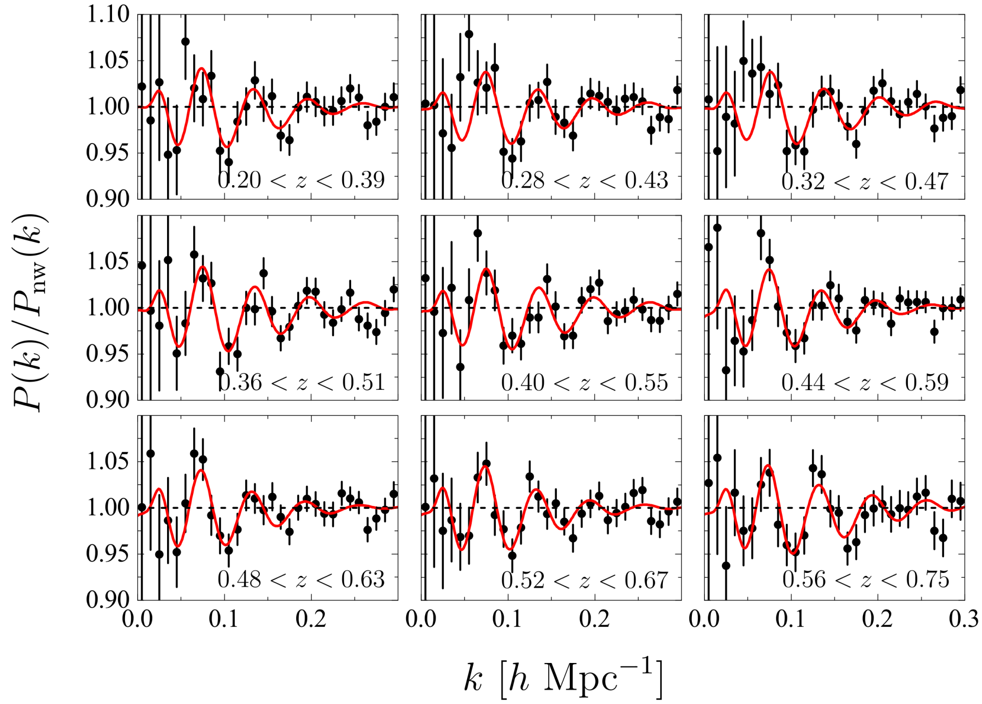


FIGURE 3. The BAO fit in $k$-space using BOSS DR12 data [6].

$$
\begin{aligned}
k' &= \frac{k(1+\epsilon)}{\alpha}\left\{1+\mu^2\left[(1+\epsilon)^{-6}-1\right]\right\}^{1/2} \\
\mu' &= \frac{\mu}{(1+\epsilon)^3}\left\{1+\mu^2\left[(1+\epsilon)^{-6}-1\right]\right\}^{-1/2}
\end{aligned} \tag{13}
$$

where

$$\alpha = \alpha_\perp^{2/3}\alpha_\parallel^{1/3}, \quad 1+\epsilon = \left(\frac{\alpha_\parallel}{\alpha_\perp}\right)^{1/3} \tag{14}$$

## 2.2. **BAO using correlation function multipoles.**

$$P_\ell(k) = \frac{2\ell+1}{2}\int_{-1}^{1} P(k,\mu)\mathcal{L}_\ell(\mu)d\mu \tag{15}$$

$$\xi_\ell(s) = \frac{i^\ell}{2\pi^2}\int k^2 P_\ell(k) j_\ell(ks)dk \tag{16}$$

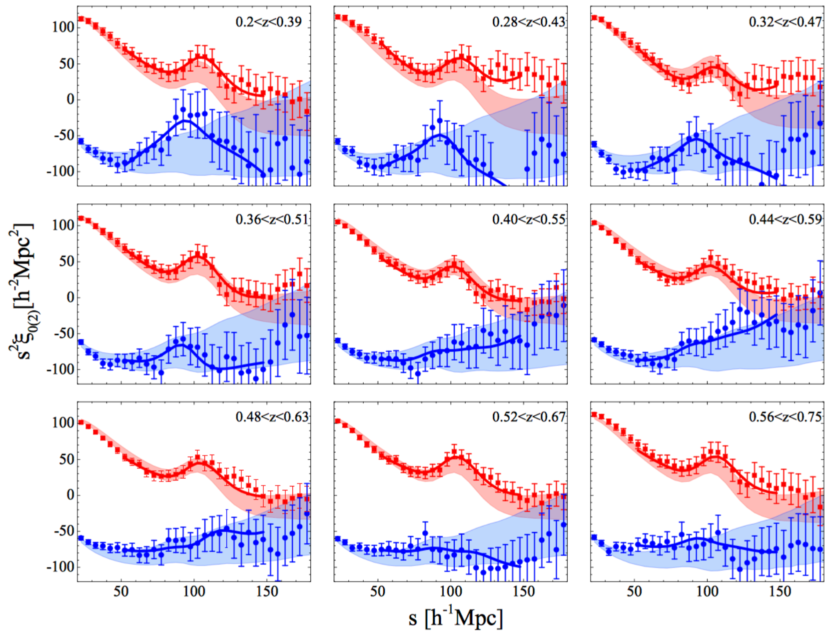


FIGURE 4. The BAO fit in $s$-space using BOSS DR12 data [7].

$$\xi(s,\mu) = \sum_{\ell} \xi_\ell(s)\mathcal{L}_\ell(\mu) \tag{17}$$

$$\xi_\ell\left(s,\alpha_\perp,\alpha_\parallel\right) = \frac{2\ell+1}{2}\int_{-1}^{1} \xi\left(s',\mu'\right)\mathcal{L}_\ell(\mu)d\mu \tag{18}$$

where

$$s' = s\sqrt{\mu^2\alpha_\parallel^2 + (1-\mu^2)\,\alpha_\perp^2};\ \mu' = \mu\alpha_\parallel/\sqrt{\mu^2\alpha_\parallel^2 + (1-\mu^2)\,\alpha_\perp^2} \tag{19}$$

$$A_\ell(s) = \frac{a_{\ell,1}}{s^2} + \frac{a_{\ell,2}}{s} + a_{\ell,3} \tag{20}$$

$$
\begin{aligned}
\xi_0^{\mathrm{mod}}(s) &= B_0 \xi_0\left(s, \alpha_\perp, \alpha_\parallel\right) + A_0(s) \\
\xi_2^{\mathrm{mod}}(s) &= \xi_2\left(s, \alpha_\perp, \alpha_\parallel\right) + A_2(s)
\end{aligned} \tag{21}
$$

With this methodology, the BAO distance has been well measured from SDSS, 2dF [2, 3, 4, 5, 6, 7, 8] and DESI surveys [9, 10, 11], especially with the BAO-reconstruction technique [12, 13].

## References


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