# LECTURE 12: THE MEASUREMENT OF POWER SPECTRUM AND CORRELATION FUNCTION

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## 1. The FKP weight

The FKP weight is the optimal weight for measurement of power spectrum multipoles [1].

$$P(k)=P(\mathbf{k}) \equiv \int d^3 r \xi(\mathbf{r}) e^{i \mathbf{k} \cdot \mathbf{r}} \tag{1}$$

$$F(\mathbf{r}) \equiv \frac{w(\mathbf{r})\left[n_g(\mathbf{r})-\alpha n_s(\mathbf{r})\right]}{\left[\int d^3 r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})\right]^{1/2}} \tag{2}$$

$$\left\langle|F(\mathbf{k})|^2\right\rangle=\frac{\int d^3 r \int d^3 r^{\prime} w(\mathbf{r}) w\left(\mathbf{r}^{\prime}\right)\left\langle\left[n_g(\mathbf{r})-\alpha n_s(\mathbf{r})\right]\left[n_g\left(\mathbf{r}^{\prime}\right)-\alpha n_s\left(\mathbf{r}^{\prime}\right)\right]\right\rangle e^{i \mathbf{k} \cdot\left(\mathbf{r}-\mathbf{r}^{\prime}\right)}}{\int d^3 r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})}$$

$$\begin{aligned}
&\left\langle n_g(\mathbf{r}) n_g\left(\mathbf{r}^{\prime}\right)\right\rangle=\bar{n}(\mathbf{r}) \bar{n}\left(\mathbf{r}^{\prime}\right)\left[1+\xi\left(\mathbf{r}-\mathbf{r}^{\prime}\right)\right]+\bar{n}(\mathbf{r}) \delta\left(\mathbf{r}-\mathbf{r}^{\prime}\right) \\
&\left\langle n_s(\mathbf{r}) n_s\left(\mathbf{r}^{\prime}\right)\right\rangle=\alpha^{-2} \bar{n}(\mathbf{r}) \bar{n}\left(\mathbf{r}^{\prime}\right)+\alpha^{-1} \bar{n}(\mathbf{r}) \delta\left(\mathbf{r}-\mathbf{r}^{\prime}\right) \\
&\left\langle n_g(\mathbf{r}) n_s\left(\mathbf{r}^{\prime}\right)\right\rangle=\alpha^{-1} \bar{n}(\mathbf{r}) \bar{n}\left(\mathbf{r}^{\prime}\right)
\end{aligned} \tag{3}$$

$$\left\langle|F(\mathbf{k})|^2\right\rangle=\int \frac{d^3 k^{\prime}}{(2 \pi)^3} P\left(\mathbf{k}^{\prime}\right)\left|G\left(\mathbf{k}-\mathbf{k}^{\prime}\right)\right|^2+(1+\alpha) \frac{\int d^3 r \bar{n}(\mathbf{r}) w^2(\mathbf{r})}{\int d^3 r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})} \tag{4}$$

$$G(\mathbf{k}) \equiv \frac{\int d^3 r \bar{n}(\mathbf{r}) w(\mathbf{r}) e^{i \mathbf{k} \cdot \mathbf{r}}}{\left[\int d^3 r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})\right]^{1/2}} \tag{5}$$

$$\left\langle|F(\mathbf{k})|^2\right\rangle \simeq P(\mathbf{k})+P_{\text {shot }} \tag{6}$$

$$P_{\text {shot }} \equiv \frac{(1+\alpha) \int d^3 r \bar{n}(\mathbf{r}) w^2(\mathbf{r})}{\int d^3 r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})} \tag{7}$$

$$\hat{P}(\mathbf{k})=|F(\mathbf{k})|^2-P_{\text {shot }} \tag{8}$$

$$\hat{P}(k) \equiv \frac{1}{V_k} \int_{V_k} d^3k' \hat{P}\left(\mathbf{k}'\right) \tag{9}$$

$$\sigma_P^2(k) \simeq \frac{1}{V_k} \int d^3k' \left| P(k) Q\left(\mathbf{k}'\right) + S\left(\mathbf{k}'\right)\right|^2 \tag{10}$$

where

$$Q(\mathbf{k}) \equiv \frac{\int d^3r \overline{n}^2(\mathbf{r}) w^2(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}}{\int d^3r \overline{n}^2(\mathbf{r}) w^2(\mathbf{r})} \tag{11}$$

and

$$S(\mathbf{k}) \equiv \frac{(1+\alpha) \int d^3r \overline{n}(\mathbf{r}) w^2(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}}{\int d^3r \overline{n}^2(\mathbf{r}) w^2(\mathbf{r})} \tag{12}$$

$$\frac{\sigma_P^2(k)}{P^2(k)} = \frac{(2\pi)^3 \int d^3r \overline{n}^4 w^4 [1 + 1/\overline{n}P(k)]^2}{V_k \left[\int d^3r \overline{n}^2 w^2\right]^2} \tag{13}$$

The KFP weight taking the following form minimises Eq (13):

$$w_0(\mathbf{r}) = \frac{1}{1 + \overline{n}(\mathbf{r}) P(k)} \tag{14}$$

## 2. The measurement of the anisotropic correlation function

Pair counts in 'data' are compared to pair counts in 'random' samples that follow the geometry of the survey. We assume a catalogue of $n_d$ objects in the data sample and $n_r$ in the random sample and then calculate three sets of numbers of pairs as a function of the binned comoving separation $s$,

- Data-data pairs $dd(s)$ that normalized by the total number of data pairs:

$$DD(s) = \frac{dd(s)}{n_d\left(n_d - 1\right)/2}; \tag{15}$$

- Random-Random pairs $rr(s)$ that normalized by the total number of random pairs:

$$RR(s) = \frac{rr(s)}{n_r\left(n_r - 1\right)/2}; \tag{16}$$

- Data-Random pairs $dr(s)$ that normalized by the total number of cross pairs:

$$DR(s) = \frac{dr(s)}{n_r n_d}. \tag{17}$$

The most commonly used Landy & Szalay estimator [4]:

$$\hat{\xi}(s) = \frac{DD - 2DR + RR}{RR}. \tag{18}$$

The task now is to compute the pair counts quickly for a large sample, which is challenging. For example, for BOSS DR12 sample, there are $\sim$ 1M data samples, and $\sim$ 100M

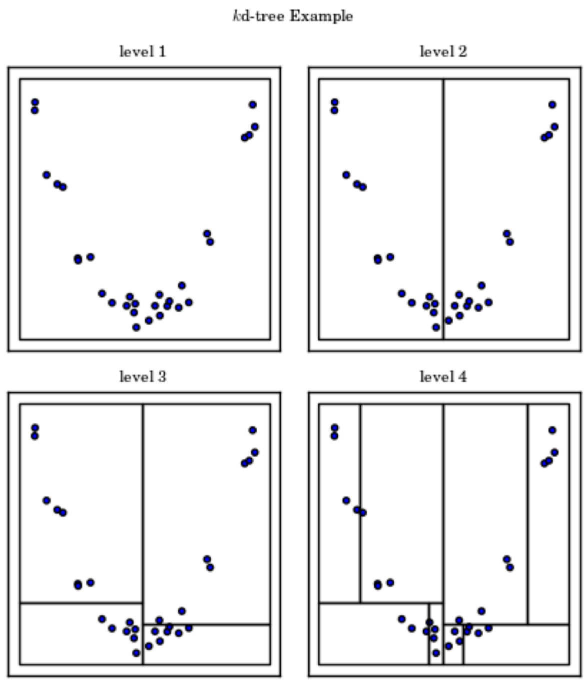


FIGURE 1. An example of kdtree.

random samples! So this is mission impossible for a brutal force algorithm with $O(N^2)$ complexity.

The way out is to use the kd-tree algorithm [1].

The kdtree code for the measurement: `https://github.com/wll745881210/KDTPCF`

[1] `http://www.linuxclustersinstitute.org/conferences/archive/2008/PDF/Dolence_98279.pdf`

```
function dualtree(node1, node2) {
  If node1 and node2 have already been compared in reverse
    return
  Let dmin = minimum distance between node1 and node2
  If dmin is greater than maximum distance of interest
    return
  Let dmax = maximum distance between node1 and node2
  Let binmin = distance bin for distance dmin
  Let binmax = distance bin for distance dmax
  If binmin = binmax
    Add node1.cnt x node2.cnt to bin count
  Else if node1 and node2 are leaf nodes
    For all points i owned by node1
      Let smin = minimum distance between point i and node2
      Let smax = maximum distance between point i and node2
        Let binmin = distance bin for smin
        Let binmax = distance bin for distance smax
        If binmin = binmax
          Add node2.cnt to bin count
        Else
          Find distances to all points in node2 from point i
          Find distance bins and update the bin counts
    Else
      If node1.cnt > node2.cnt
        dualtree(node1.leftChild, node2)
        dualtree(node1.rightChild, node2)
      Else
        dualtree(node1, node2.leftChild)
        dualtree(node1, node2.rightChild)
  }
```

FIGURE 2. The pseudo-code of kdtree.

## References

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