LECTURE 10: BAO, RSD AND THE AP EFFECT (II)

GONG-BO ZHAO

1. Redshift Space Distortions

RSD is another special three-dimensional clustering pattern of galaxies, but it is due to local motions of the galaxies under gravity [3]. Suppose galaxies only co-move with the cosmic background without moving locally, the clustering of galaxies should be isotropic, in other words, there are same number of galaxy pairs along or across lines of sight of the observer. However, this is not the case in the real world. Galaxies move towards nearby galaxies due to gravity, thus they have "peculiar velocities". As we infer galaxies' positions from their line-of-sight velocities in astronomy, the peculiar velocities can distort galaxies' positions, which results in a distortion, called Redshift Space Distortion (RSD), in the 3D clustering of galaxies. RSD plays a key role in cosmology as it can directly be used to infer the nature of gravity (remember the peculiar motion is caused by gravity!).

(1)
$$\begin{gathered} 1+z_{\mathrm{obs}}=\left(1+z_{\mathrm{cos}}\right)\left(1-\frac{v_{\|}(\boldsymbol{r})}{c}\right)^{-1} \\ \boldsymbol{s}=\boldsymbol{r}+\frac{\left(1+z_{\mathrm{cos}}\right) v_{\|}(\boldsymbol{r})}{H\left(z_{\mathrm{cos}}\right)} \hat{r} \end{gathered}$$

(2)
$$\rho_{\mathrm{m}}^{s}(\boldsymbol{s}) d \boldsymbol{s}=\rho_{\mathrm{m}}(\boldsymbol{r}) d \boldsymbol{r}$$

(3)
$$J=\left|\frac{d \boldsymbol{r}}{d \boldsymbol{s}}\right|=\frac{r^{2} d r}{s^{2} d s}=\left\{1-\frac{\left(1+z_{\mathrm{cos}}\right)}{H\left(z_{\mathrm{cos}}\right)} \frac{v_{\|}}{r}\right\}^{-2}\left\{1-\frac{\left(1+z_{\mathrm{cos}}\right)}{H\left(z_{\mathrm{cos}}\right)} \frac{\partial v_{\|}}{\partial r}\right\}^{-1}$$

(4)
$$J \simeq\left\{1-\frac{\left(1+z_{\mathrm{cos}}\right)}{H\left(z_{\mathrm{cos}}\right)} \frac{\partial v_{\|}}{\partial r}\right\}^{-1}$$

(5)
$$\theta(\boldsymbol{x}) \equiv-\frac{\nabla \cdot \boldsymbol{v}(\boldsymbol{x})}{a H f}$$

(6)
$$\boldsymbol{v}(\boldsymbol{k})=-i a H f \frac{\boldsymbol{k}}{k^{2}} \theta(\boldsymbol{k})$$

Kaiser formula

(Kaiser, 1987, MNRAS, 227, 1)

- Mass conservation $(1+\delta^r)\,d^3r = (1+\delta^s)\,d^3s$
- Jacobian $\frac{d^3s}{d^3r} = \left(1+\frac{v}{z}\right)^2\left(1+\frac{dv}{dz}\right)$
- Distant observer $1+\delta^s = (1+\delta^r)\left(1+\frac{dv}{dz}\right)^{-1}$
- Potential flow $\frac{dv}{dz} = -\frac{d^2}{dz^2}\nabla^{-2}\theta$
- Proportionality $\delta^s(\mathbf{k}) = \delta^r(\mathbf{k}) + \mu_k^2\theta(\mathbf{k}) \simeq \left(1+f\mu_k^2\right)\delta^r(\mathbf{k})$

$$\delta_{\rm m}^{s}(\boldsymbol{s}) = \left|\frac{d\boldsymbol{s}}{d\boldsymbol{r}}\right|^{-1}\{1+\delta_{\rm m}(\boldsymbol{r})\} - 1 \tag{7}$$

$$\delta_{\rm m}^{s}(\boldsymbol{k}) = \int d^3x \left\{\delta_{\rm m}(\boldsymbol{x}) - \frac{1}{aH}\frac{\partial v_z(\boldsymbol{x})}{\partial z}\right\} e^{i\boldsymbol{k}\cdot\boldsymbol{x}+ik\mu v_z/(aH)} \tag{8}$$

$$\begin{aligned}
\delta_{\rm m}^{s,L}(\boldsymbol{k}) &= \delta_{\rm m}(\boldsymbol{k}) - \int d^3x e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\frac{1}{aH}\frac{\partial}{\partial z}\int\frac{d^3k'}{(2\pi)^3}e^{-i\boldsymbol{k}'\cdot\boldsymbol{x}}v_z\left(\boldsymbol{k}'\right)\\
&= \delta_{\rm m}(\boldsymbol{k}) + f\int\frac{d^3k'}{(2\pi)^3}\int d^3x e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{x}}\frac{k_zk'_z}{k^2}\theta(\boldsymbol{k})\\
&= \delta_{\rm m}(\boldsymbol{k}) + f\mu^2\theta(\boldsymbol{k})\\
&= \left(1+f\mu^2\right)\delta_{\rm m}^{L}(\boldsymbol{k})
\end{aligned} \tag{9}$$

The famous Kaiser formula:

$$P_g(k) = (b+f\mu^2)^2P_m(k) \tag{10}$$

where P_g and P_m are galaxy power spectrum in the redshift space and the matter power spectrum in real space, respectively. For a proof, see Sec. 9.4 in Dodelson's book.

2. Revisiting the Alcock-Paczynski (AP) effect

The Alcock-Paczynski (AP) effect [1] is another kind of geometric distortion similar to the RSD, but due to completely different physics. Remember in cosmology we use redshifts to get distances, thus this process is cosmology-dependent. If we use a wrong cosmology to get the distances, which we inevitably do, we get wrong distances on both radial and transverse directions. The errors we get on these two directions are different,

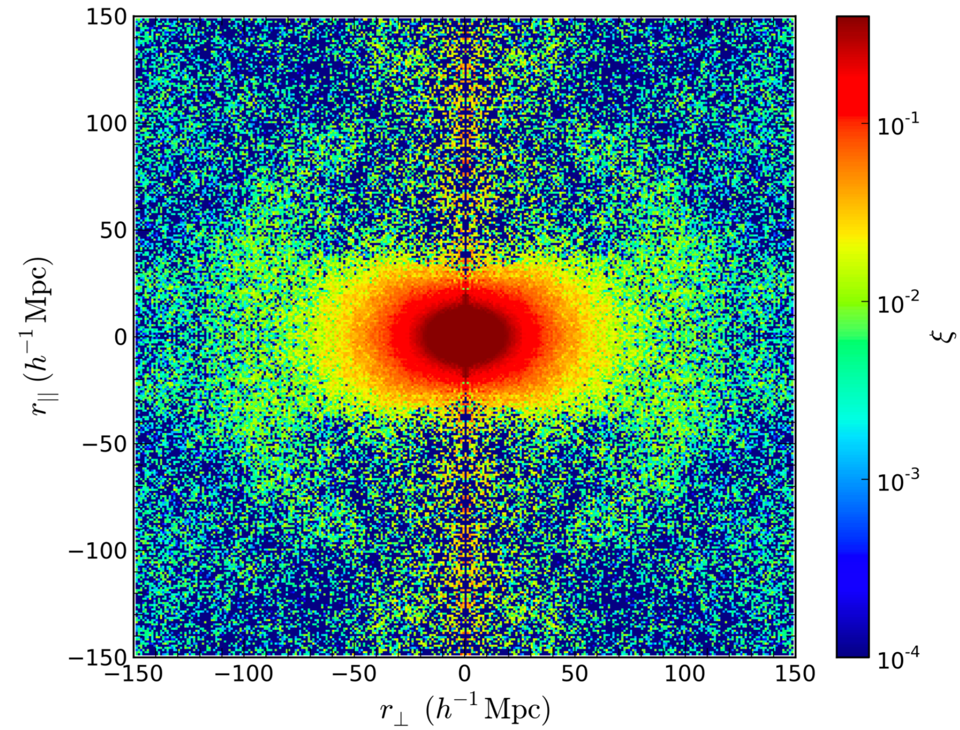

Figure 1. The 2D correlation function measured from the BOSS galaxies [5].

thus a distortion is caused. This effect can be used to probe the geometry of the Universe through measurements of D_A and H.

$$\alpha_\parallel = \frac{H^{\rm fid}(z) r_s^{\rm fid}(z_d)}{H(z) r_s(z_d)}$$
$$\alpha_\perp = \frac{D_A(z) r_s^{\rm fid}(z_d)}{D_A^{\rm fd}(z) r_s(z_d)} \tag{11}$$

Key steps:

$$\mathbf{x}' = S\cdot\mathbf{x} \quad ; \quad \mathbf{k}' = S^{-1}\cdot\mathbf{k} \tag{12}$$

$$S = \begin{pmatrix} \alpha_\perp^{-1} & 0 & 0 \\ 0 & \alpha_\perp^{-1} & 0 \\ 0 & 0 & \alpha_\parallel^{-1} \end{pmatrix} \tag{13}$$

$$P(\mathbf{k}) = \int \xi(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} d^3 r \tag{14}$$

$$P'\left(\mathbf{k}'\right) = \int \xi(\mathbf{r}) e^{i(S\cdot\mathbf{k}')\cdot\mathbf{r}} |S| d^3 r = |S| P\left(S\cdot\mathbf{k}'\right) \tag{15}$$

$$P'\left(k'_{\|}, \mathbf{k}'_{\perp}\right) = \frac{1}{\alpha_{\perp}^2 \alpha_{\|}} P\left(\frac{k'_{\|}}{\alpha_{\|}}, \frac{\mathbf{k}'_{\perp}}{\alpha_{\perp}}\right) \tag{16}$$

$$\begin{aligned} P\left(k_{\|}, \mathbf{k}_{\perp}\right) &= P_0(k)\left(1+\beta\mu^2\right)^2 D\left(k\mu\sigma_{\mathrm{p}}\right) \\ &= P_0(k) k^{-4}\left[k_{\perp}^2 + (\beta+1) k_{\|}^2\right] D\left(k_{\|}\sigma_{\mathrm{p}}\right) \end{aligned} \tag{17}$$

where $D\left(k'\mu'\sigma'_{\mathrm{p}}\right) = \frac{1}{1+\left(k'\mu'\sigma'_{\mathrm{p}}\right)^2/2}$.

$$\begin{aligned} P'\left(\mathbf{k}'\right) = & \frac{1}{\alpha_{\perp}^2\alpha_{\|}} P_0\left(\sqrt{\frac{k'^2_{\perp}}{\alpha^2_{\perp}} + \frac{k'^2_{\|}}{\alpha^2_{\|}}}\right)\left(\frac{k'^2_{\perp}}{\alpha^2_{\perp}} + \frac{k'^2_{\|}}{\alpha^2_{\|}}\right)^{-2} \\ & \times\left[\frac{k'^2_{\perp}}{\alpha^2_{\perp}} + (\beta+1)\frac{k'^2_{\|}}{\alpha^2_{\|}}\right] D\left(\frac{k_{\|}\sigma_{\mathrm{p}}}{\alpha_{\|}}\right) \end{aligned} \tag{18}$$

where $k'_{\|} = \mu' k'$.

$$\begin{aligned} P'\left(\mathbf{k}'\right) = & \frac{1}{\alpha_{\perp}^2\alpha_{\|}} P_0\left[\frac{k'}{\alpha_{\perp}}\sqrt{1+\mu'^2\left(\frac{1}{F^2}-1\right)}\right] \\ & \times\left[1+\mu'^2\left(\frac{1}{F^2}-1\right)\right]^{-2} \\ & \times\left[1+\mu'^2\left(\frac{\beta+1}{F^2}-1\right)\right]^2 D\left(k'\mu'\sigma'_{\mathrm{p}}\right) \end{aligned} \tag{19}$$

if we set,

$$\begin{aligned} k' &= \frac{k}{\alpha_{\perp}}\left[1+\mu^2\left(\frac{1}{F^2}-1\right)\right]^{1/2} \\ \mu' &= \frac{\mu}{F}\left[1+\mu^2\left(\frac{1}{F^2}-1\right)\right]^{-1/2} \end{aligned} \tag{20}$$

where $F = \alpha_{\|}/\alpha_{\perp}$.

For multipoles,

$$P_\ell(k) = \left(\frac{r_s^{\rm fid}}{r_s}\right)^3 \frac{(2\ell+1)}{2\alpha_\perp^2\alpha_\parallel} \int_{-1}^{1} d\mu P_{\rm g}\left[k'(k,\mu),\mu'(\mu)\right]\mathcal{L}_\ell(\mu) \tag{21}$$

For details, see [2].

References

[1] C. Alcock and B. Paczynski, Nature **281**, 358 (1979). doi:10.1038/281358a0
[2] W. E. Ballinger, J. A. Peacock and A. F. Heavens, Mon. Not. Roy. Astron. Soc. **282**, 877 (1996) doi:10.1093/mnras/282.3.877 [astro-ph/9605017].
[3] N. Kaiser, Mon. Not. Roy. Astron. Soc. **227**, 1 (1987).
[4] J. A. Peacock *et al.*, Nature **410**, 169 (2001) doi:10.1038/35065528 [astro-ph/0103143].
[5] L. Samushia *et al.*, Mon. Not. Roy. Astron. Soc. **439**, no. 4, 3504 (2014) doi:10.1093/mnras/stu197 [arXiv:1312.4899 [astro-ph.CO]].