LECTURE 13: THE MEASUREMENT OF BAO AND RSD

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1. The survey window function

Due to the geometry of the survey volume, we have to convolve the theoretical power spectrum with the survey window function, which is a pain, but there is no way around it.

$$\delta'(\mathbf{x}) = \delta(\mathbf{x})W(\mathbf{x}), \tag{1}$$

$$\tilde{\delta}'(\mathbf{k}) = \tilde{\delta}(\mathbf{k}) * \tilde{W}(\mathbf{k}), \tag{2}$$

where $*$ denotes a convolution.

$$P'(\mathbf{k}) = \int \frac{d^3q}{(2\pi)^3} P(\mathbf{k}-\mathbf{q})|\tilde{W}(\mathbf{q})|^2 \tag{3}$$

Fortunately, it was found that the 3D convolution can be broken into 1D Hankel transformations, due to the convolution theorem [1].

$$P_\ell(k) = 4\pi(-i)^\ell \int \Delta^2 d\Delta \xi_\ell(\Delta) j_\ell(k\Delta) \tag{4}$$

$$P'_\ell(k) = 4\pi(-i)^\ell \left(\frac{2\ell+1}{2q+1}\right) \times A^q_{\ell,\ell'} \int \Delta^2 d\Delta \xi_{\ell'}(\Delta) Q_q(\Delta) j_\ell(k\Delta) \tag{5}$$

$$RR_q^{\rm tot}(\Delta) = \frac{1}{2}\overline{n}_s^2 2\pi\Delta^3 d(\ln\Delta) Q_q(\Delta) \tag{6}$$

$$\xi'_0(\Delta) = \xi_0 Q_0 + \frac{1}{5}\xi_2 Q_2 + \frac{1}{9}\xi_4 Q_4 + \frac{1}{13}\xi_6 Q_6 + \cdots \tag{7}$$

$$\begin{aligned} \xi'_2(\Delta) = \xi_0 Q_2 + \xi_2 & \left(Q_0 + \frac{2}{7}Q_2 + \frac{2}{7}Q_4\right) \\ + \xi_4 & \left(\frac{2}{7}Q_2 + \frac{100}{693}Q_4 + \frac{25}{143}Q_6\right) \\ + \xi_6 & \left(\frac{25}{143}Q_4 + \frac{14}{143}Q_6 + \frac{28}{221}Q_8\right) \end{aligned} \tag{8}$$

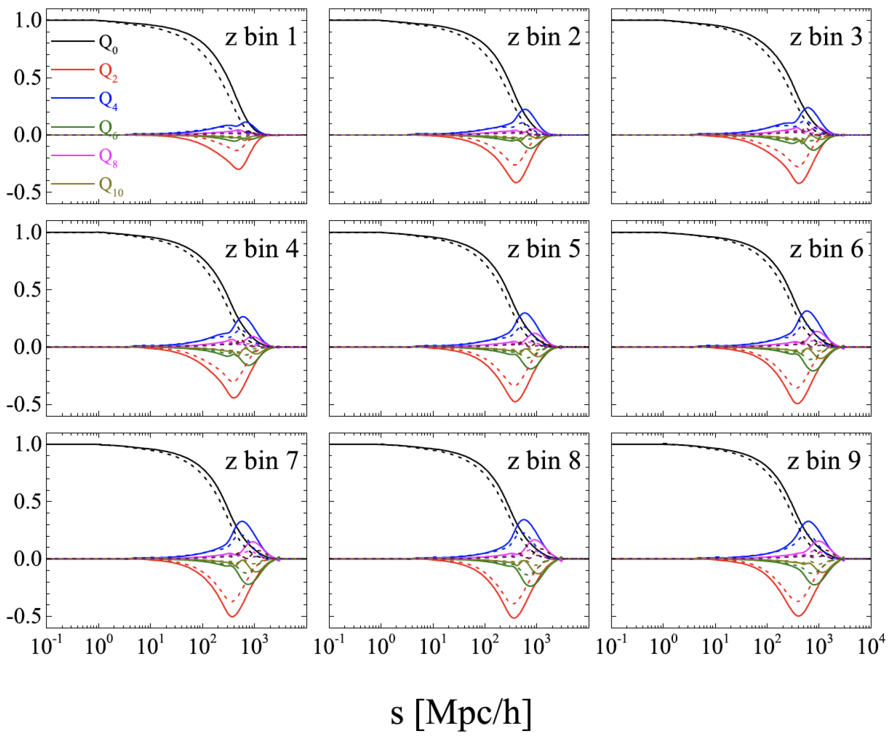

FIGURE 1. The survey window function for BOSS DR12 [2].

2. THE MEASUREMENT OF BAO

2.1. BAO using power spectrum multipoles. The template:

$$\alpha_\perp = \frac{D_A(z) r_d^{\rm fid}}{D_A^{\rm fid}(z) r_d}, \quad \alpha_\parallel = \frac{H^{\rm fid}(z) r_d^{\rm fid}}{H(z) r_d} \tag{9}$$

$$P_g(k,\mu) = P_{\rm nw}(k,\mu)\left\{1 + O(k) e^{-k^2\left[\mu^2\Sigma_\parallel^2 + (1-\mu^2)\Sigma_\perp^2\right]/2}\right\} \tag{10}$$

$$P_{\rm nw}(k,\mu) = B^2\left(1+\beta\mu^2\right)^2 P_{\rm nw,lin}(k) F(k,\mu) \tag{11}$$

$$F(k,\mu) = \frac{1}{(1 + k^2\mu^2\Sigma_s^2/2)} \tag{12}$$

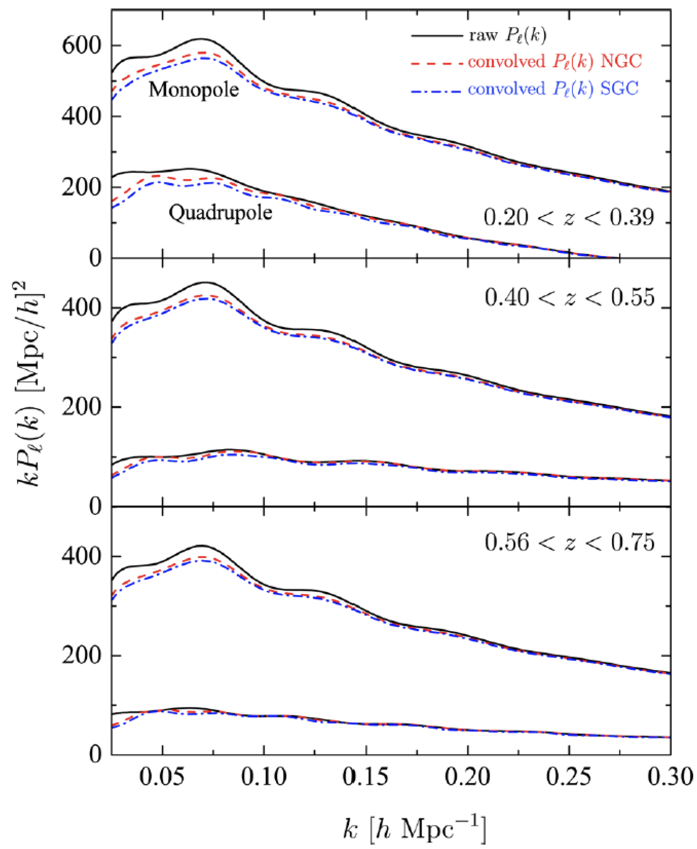

FIGURE 2. The convolved power spectra [2].

$$P_\ell(k) = \left(\frac{r_s^{\mathrm{fid}}}{r_s}\right)^3 \frac{2\ell+1}{2\alpha_\perp^2\alpha_\parallel} \int_{-1}^{1} \mathrm{d}\mu P_g\left(k', \mu'\right) \mathcal{L}_\ell(\mu) + \frac{a_{\ell 1}}{k^3} + \frac{a_{\ell 2}}{k^2} + \frac{a_{\ell 3}}{k} + a_{\ell 4} + a_{\ell 5} k$$

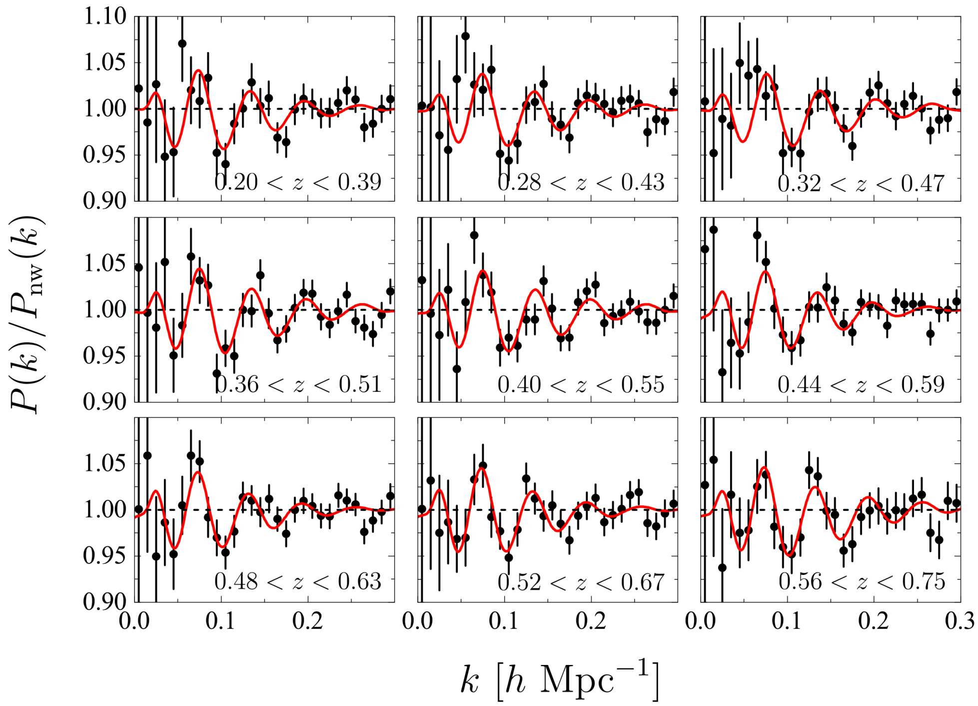

FIGURE 3. The BAO fit in k-space using BOSS DR12 data [2].

$$
\begin{aligned}
k' &= \frac{k(1+\epsilon)}{\alpha}\left\{1+\mu^2\left[(1+\epsilon)^{-6}-1\right]\right\}^{1/2} \\
\mu' &= \frac{\mu}{(1+\epsilon)^3}\left\{1+\mu^2\left[(1+\epsilon)^{-6}-1\right]\right\}^{-1/2}
\end{aligned} \tag{13}
$$

where

$$
\alpha = \alpha_\perp^{2/3}\alpha_\parallel^{1/3}, \quad 1+\epsilon = \left(\frac{\alpha_\parallel}{\alpha_\perp}\right)^{1/3} \tag{14}
$$

2.2. **BAO using correlation function multipoles.**

$$
P_\ell(k) = \frac{2\ell+1}{2}\int_{-1}^{1} P(k,\mu)\mathcal{L}_\ell(\mu)d\mu \tag{15}
$$

$$
\xi_\ell(s) = \frac{i^\ell}{2\pi^2}\int k^2 P_\ell(k) j_\ell(ks)dk \tag{16}
$$

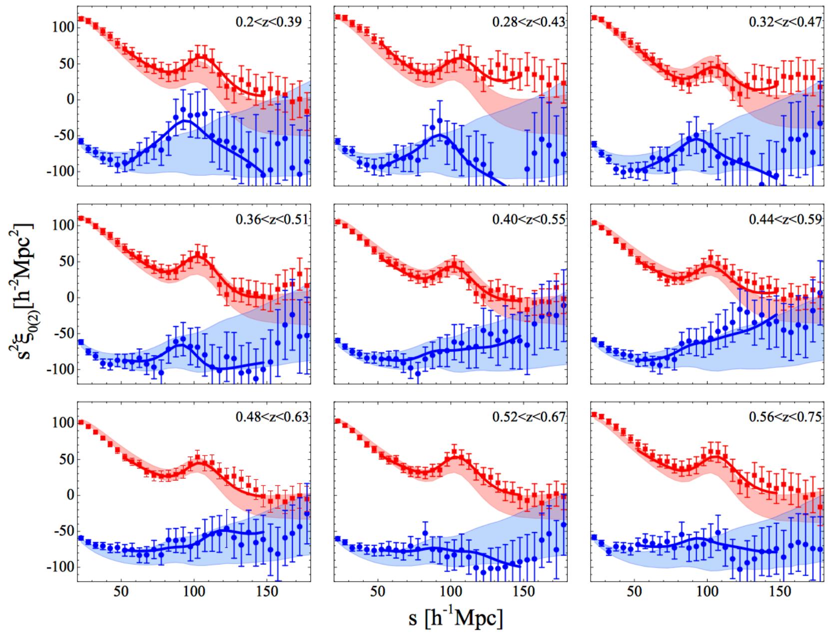

FIGURE 4. The BAO fit in s-space using BOSS DR12 data [3].

$$\xi(s,\mu) = \sum_{\ell} \xi_\ell(s)\mathcal{L}_\ell(\mu) \tag{17}$$

$$\xi_\ell\left(s,\alpha_\perp,\alpha_\parallel\right) = \frac{2\ell+1}{2}\int_{-1}^{1}\xi\left(s',\mu'\right)\mathcal{L}_\ell(\mu)d\mu \tag{18}$$

where

$$s' = s\sqrt{\mu^2\alpha_\parallel^2 + (1-\mu^2)\,\alpha_\perp^2};\ \mu' = \mu\alpha_\parallel/\sqrt{\mu^2\alpha_\parallel^2 + (1-\mu^2)\,\alpha_\perp^2} \tag{19}$$

$$A_\ell(s) = \frac{a_{\ell,1}}{s^2} + \frac{a_{\ell,2}}{s} + a_{\ell,3} \tag{20}$$

$$\xi_0^{\mathrm{mod}}(s) = B_0\xi_0\left(s, \alpha_\perp, \alpha_\parallel\right) + A_0(s)$$
$$\xi_2^{\mathrm{mod}}(s) = \xi_2\left(s, \alpha_\perp, \alpha_\parallel\right) + A_2(s) \tag{21}$$

3. The measurement of RSD in k-space

The RSD is much more difficult to measure accurately, simply because it is harder to model the velocity power spectrum properly, even on linear scales. The commonly used model, or template, of the RSD is called the TNS model [4].

$$\begin{aligned} P_{\mathrm{g}}(k, \mu, z) =& D_{\mathrm{FOG}}\left(k, \mu_2, z\right)\left[P_{\mathrm{g},\delta\delta}(k, z)\right. \\ &+ 2f(z)\mu^P P_{\mathrm{g},\delta\theta}(k, z) + f^2(z)\mu^4 P_{\theta\theta}(k, z) \\ &\left.+ A(k, \mu, z) + B(k, \mu, z)\right] \end{aligned} \tag{22}$$

where,

$$\begin{aligned} P_{\mathrm{g},\delta\delta}(k, z) =& b_1^2(z)P_{\delta\delta}(k, z) + 2b_1(z)b_2(z)P_{\mathrm{b2},\delta}(k, z) \\ &- \frac{8}{7}b_1(z)\left(b_1(z) - 1\right)P_{\mathrm{bs2},\delta}(k, z) \\ &+ \frac{64}{315}b_1(z)\left(b_1(z) - 1\right)\sigma_3^2(k, z)P_m^{\mathrm{L}}(k, z) \\ &+ b_2^2(z)P_{\mathrm{b22}}(k, z) - \frac{8}{7}\left[b_1(z) - 1\right]b_2(z)P_{\mathrm{b2s2}}(k, z) \\ &+ \frac{16}{49}\left[b_1(z) - 1\right]^2 P_{\mathrm{bs2}}(k, z) \end{aligned} \tag{23}$$

$$\begin{aligned} P_{\mathrm{g},\delta\theta}(k, z) =& b_1(z)P_{\delta\theta}(k, z) + b_2(z)P_{\mathrm{b2},\theta}(k, z) \\ &- \frac{4}{7}\left[b_1(z) - 1\right]P_{\mathrm{bs2},\theta}(k, z) \\ &+ \frac{32}{315}\left[b_1(z) - 1\right]\sigma_3^2(k, z)P_m^{\mathrm{L}}(k, z) \end{aligned} \tag{24}$$

$$\begin{aligned} P_{\mathrm{g},\theta\theta}(k, z) &= P_{\theta\theta}(k, z) \\ D_{\mathrm{FoG}}(k, \mu, z) &= \left\{1 + \left[k\mu\sigma_v(z)\right]^2/2\right\}^{-2} \\ A(k, \mu, z) &= b_1^3(z)\sum_{m,n=1}^{3}\mu^{2m}\left[f(z)/b_1(z)\right]^n A_{mn}(k, z) \\ B(k, \mu, z) &= b_1^4(z)\sum_{m=1}^{4}\sum_{a,b=1}^{2}\mu^{2m}\left[-f(z)/b_1(z)\right]^{a+b}B_{ab}^m(k, z) \end{aligned} \tag{25}$$

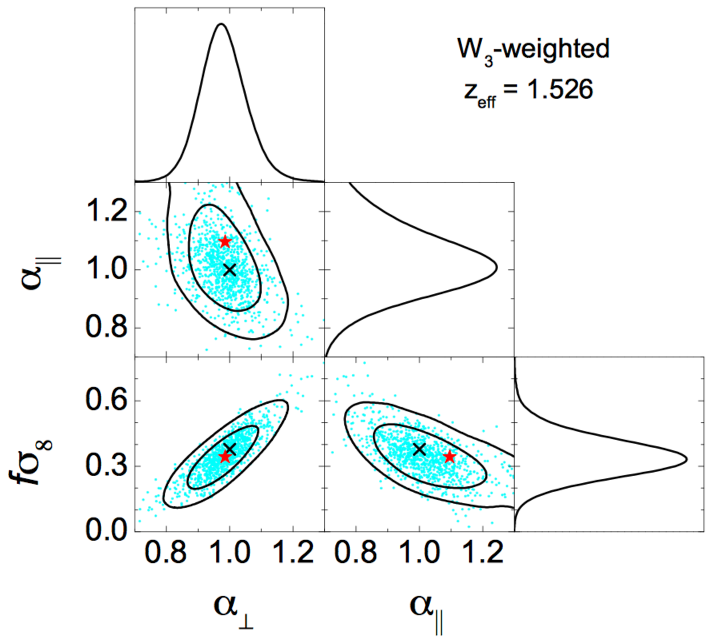

FIGURE 5. The BAO and RSD fit in k-space using BOSS DR14 quasar mock data [5].

$$\begin{aligned} b_{\mathrm{s2}} &= -\frac{4}{7}(b_1 - 1) \\ b_{\mathrm{3nl}} &= \frac{32}{315}(b_1 - 1) \end{aligned} \tag{26}$$

REFERENCES

[1] M. J. Wilson, J. A. Peacock, A. N. Taylor and S. de la Torre, Mon. Not. Roy. Astron. Soc. **464**, no. 3, 3121 (2017) doi:10.1093/mnras/stw2576 [arXiv:1511.07799 [astro-ph.CO]].

[2] G. B. Zhao *et al.* [BOSS Collaboration], Mon. Not. Roy. Astron. Soc. **466**, no. 1, 762 (2017) doi:10.1093/mnras/stw3199 [arXiv:1607.03153 [astro-ph.CO]].

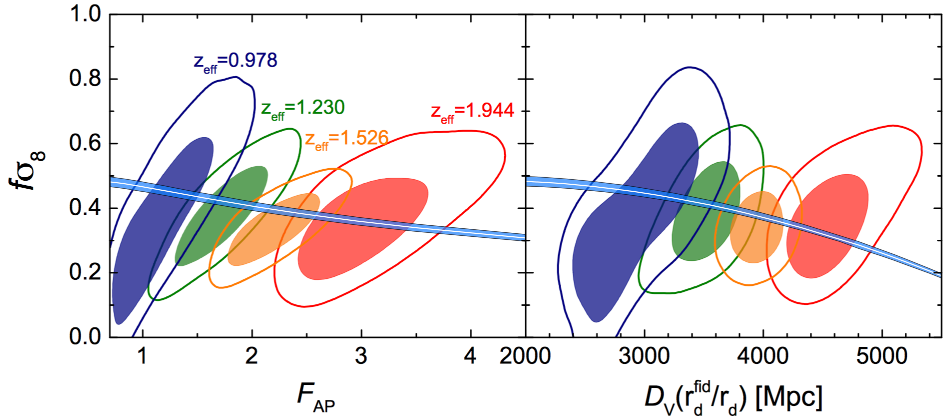

FIGURE 6. The BAO and RSD fit in k-space using the actual BOSS DR14 quasar data [5].

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